CASE STUDY ANALYSIS OF MATHEMATICS LITERACY WORKERS' IDENTITY AND

UNDERSTANDING OF NUMBERS WITHIN A COMMUNITY OF PRACTICE

by

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(Under the Direction of Dorothy Y. White)

ABSTRACT

The purpose of this study was to examine how the experiences of African American college mathematics literacy workers (CMLWs) and high school mathematics literacy workers (MLWs), within a community of practice namely the Young People's Project (YPP) Chicago, influence their identities. This study also examined the mathematics strategies and understanding of number concepts participants had as a result of their involvement in a Flagway Game workshop training. The YPP is a youth empowerment and after-school mathematics initiative created by students in Jackson, Mississippi. The Flagway Game was a number theory game derived from the Möbius Function, to help students expand their understanding of natural numbers. The focus of this study was an MLW Flagway Game workshop training, which took place over 4 weeks at Abelin High in January 2007.

The study used interpretive qualitative case study methodology (Yin, 2003), grounded within a theoretical framework of Wenger's (1998) communities of practice (CoPs). Interviews were conducted with participants where they were asked to complete mathematical tasks to determine their levels of understanding of some number concepts used in Flagway. Participants were also asked to reflect on their experience in Flagway training and how that experience shaped their mathematics literacy work. Within-case analyses as well as cross-case analysis of participant findings were conducted. Participant responses were also analyzed using Wenger's three modes of belonging: engagement, imagination, and alignment, as well as from a critical race theory (CRT) perspective using counter-narratives, which showed how they worked towards mathematics literacy for liberation contrary to dominant narratives of failure and passivity.

There were common themes of identity across participants, which influenced how they understood number concepts. CMLWs and MLWs viewed themselves as being; role models, agents of change, and doers of mathematics. There were also common themes of mathematics strategies and understanding of numbers across participants. CMLWs and MLWs used different memorization strategies and also identified ways in which their flexibility with numbers were enhanced as result of their work with the Flagway Game. The CMLWs were more likely than the MLWs to link their practice to a broader community of mathematics literacy workers. The CMLWs structured their practice to align with future pursuits. CMLWs and MLWs used persistence and a commitment to their community in obtaining mathematics literacy for

themselves and for others.

INDEX WORDS: Mathematics Education; The Young People's Project; The Algebra Project; Mathematics Literacy; Mathematics Literacy Workers; Student Involvement in School Reform; Student Learning; Identity; Communities of Practice; Critical Race Theory

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DEDICATIO
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For my mother, thanks for giving me the opportunity to get a good education.

For my sisters Dee Dee, Katrine, and Nicole, Sis says it's possible.

For my brothers Fabian and MoMo, thanks for always being there for me no matter what.

For my nieces Nadiyah, Sanaa, Indigo and Alexis, the future is yours.

For my nephews, Bryce, Ezekiel and Michael, don't be afraid to realize your dreams.

For my booba dooba, Solomon Yao, Mommy loves
you!

To all the young people. RISE
UP!

Change begins and ends with you.

May your actions today serve as inspiration for transformation tomorrow.

Yes, you can.

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CHAPTER 1

INTRODUCTION

How the Study Began

> How can young people take ownership of math literacy and be successful in today's society? Young People's Project makes the argument that mathematics is important and the Algebra Project provides the space for young people to take that ownership. Algebra Project also believes that part of the goal is not to articulate a full vision for mathematics reform. Young people must be a part of the decision making in articulating that vision. (Karima, Math Literacy Worker)

A mathematics literacy worker at an Algebra Project community-organizing meeting in Atlanta, spoke these words in 2004. Reflecting back on that day and my notes from the meeting, two things came to mind. First, there were a number of community members joined together, discussing the mathematics education of Black children. The room was filled with community organizers, activists, school administrators, university educators, parents, and teachers. The second thing that came to mind was that there were young people there as well. The young people at this meeting were confident in their ability to perform in mathematics and saw themselves as capable of changing the way mathematics could be taught to their peers. They were also making the case for the importance of algebra and describing an organization they created to help students learn mathematics. The young people I am referring to formed the youth arm of the Algebra Project's mathematics initiative, the Young People's Project (YPP), and their work as mathematics literacy workers represented a different aspect of community outreach and

reform efforts in mathematics teaching, learning, and curriculum development. Their work represented a *youth-led* initiative to transform mathematics education.

Throughout my schooling, I was an average student in mathematics. When I went to college, even though I was somewhat apprehensive, I decided to pursue mathematics as a discipline of study. I was told repeatedly that only mathematically talented students should

pursue the discipline as a major. I decided to study mathematics anyway because I wanted to disprove that assumption. Even though I worked diligently, the mathematics was tough to grasp at times. One of the most important decisions I made while in college, which transformed my understanding and the way I thought about mathematics, was the decision to work as a peer tutor in the college's mathematics tutoring laboratory. As a peer tutor, my mathematics literacy grew over time, and my ability to conceptualize mathematics in broader ways also grew. I also had the opportunity to help students who had various difficulties. The students I worked with had a wide variety of backgrounds. They had taken courses like Algebra II, Pre-Calculus, and Advanced Placement Calculus in high school but had graduated with serious deficiencies in these subjects.

Through the practice of tutoring my peers to improve *their* understanding in mathematics, *my* confidence and understanding in mathematics shifted over time. Engaging in mathematics in a social context influenced my learning and my ability to communicate mathematical ideas to others, and that experience ultimately helped to shape my mathematics literacy and mathematics

identity. Mathematics identity, as defined by Martin (2007):

> Refers to the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person's self-understandings and how they are seen by others in the context of doing mathematics....A mathematics identity is expressed in narrative form as a *negotiated* self, is always under construction, and results from the negotiation of our own assertions and the external ascriptions of others. (p. 150)

I believe that learning is connected to identity and identity is connected to learning. Today, I continue to help others understand and engage in mathematics. My engagement in mathematics continues to influence how I see myself, deepens my mathematical content knowledge, and also creates possibilities for thinking about mathematics in new and different ways.

Reflecting on my personal growth in mathematics through interaction with others, I wanted to study young people whose primary goal was to help their peers learn mathematics. I considered questions like the following:

- How has the experience of helping others learn mathematics influenced the mathematics literacy of those doing the teaching?
- How has working with young people shaped how young people do mathematics and affected their engagement in mathematics?
- What identities do young people have who work in these contexts?

Considering the work of the YPP as a social context for engagement in mathematics, I wanted to gain a deeper understanding of the role that young people have in the mathematics teaching of their peers. As young people take on the role of teacher, facilitator, and mathematics literacy worker in their practice, I also wanted to know how they see themselves as *doers of mathematics* (Martin, 2000, 2007) and what other aspects of their identity are shaped in the context of the mathematics literacy work they participate in. YPP became the focal point for this dissertation study because its members engage in this kind of transformative work. YPP's work in many ways underscores the importance of students' critical involvement in mathematics education, from their impetus to do mathematics literacy work to their understanding of its impact in their local community. This was my initial impetus for this study.

Statement of the Problem

The Mathematics Literacy Issue

As our society grows in complexity, attributed to technological advancement, in order for all citizens, regardless of race, to compete fully in society, they must have an adequate level of mathematics literacy. Unfortunately, the gap has widened between the mathematical needs of

citizens and the mathematical capabilities of individuals (Quality Literacy Design Team, 2001). Many Black students, in particular, cannot take full advantage of careers in the sciences and technologies due to insufficient knowledge in mathematics. Mathematics literacy enables

individuals to improve their life chances in study and work. It also enables individuals to participate more fully in society by making more informed economic and political decisions. Acquiring mathematics literacy also provides individuals with the opportunity to empower themselves and to broaden their perspective and awareness of issues around them (Ernest, 2002). Some argue that the citizenship issue is embedded in the mathematics literacy issue (Kamii, 1990; Martin, 2007; R. P. Moses, 1994; Silva, Moses, Rivers, & Johnson, 1990). Citizenship refers to the rights and responsibilities of individuals participating in society for the betterment of themselves and their community. Today, mathematics literacy has become a question of citizenship for many poor and minority students in the same way that having literacy and a modest interpretation of the Constitution was a requirement for citizenship for poor Mississippi sharecroppers accessing the right to vote in the 1960s (R. P. Moses, 1994; R. P. Moses & Cobb, 2001). But even then, there was a necessity for those who were denied access to the political system to make the case for themselves that they too could articulate their position by demanding their right to vote. Today, a similar articulation is necessary for Black students in mathematics. These students must make the case for themselves that acquiring mathematics literacy is not only a citizenship issue, but also a *civil rights issue* (R. P. Moses, 1994; R. P. Moses & Cobb, 2001). Further, it should be the right of every citizen to be able to access quality mathematics education to become mathematically literate. When we prepare students for citizenship, we are preparing them to take a position on issues and utilize their voices effectively in dealing with these issues and beyond (Rudduck, 2007).

The issue of Black student achievement has received an unprecedented focus in educational research (Comeaux & Jayakumar, 2006; King, 2005; Ladson-Billings, 1994, 1995, 2000b; Ogbu, 2003) in general and mathematics education (Anderson, 1990; Hughes, 2003; Johnson, 1984; Lubenski, 2002; Martin, 2000; Rech & Harrington, 2000; Secada, 1992; Stiff & Harvey, 1988; Tate, 1997a) in particular. One aspect of the problem has been the challenge for many Black students to demonstrate a competency in mathematics, not only in schools, but also on standardized tests. Many Black students have struggled to develop mathematics literacy skills that will enable them to perform well on national assessments. Data from a recent National Assessment of Educational Progress (NAEP) report on mathematics assessment showed that the average 12^{th} grade score for Black students was 127, as compared to the national average of 150 (National Center for Education Statistics, 2005a). The NAEP assessment tested students' mathematics proficiency on topics ranging from number properties and operations, measurement, geometry, and algebra. Despite moderate gains on NAEP assessments over the past 30 years, Black students still fare poorly in comparison to other racial and ethnic groups (NCES, 2000). Furthermore, evidence suggests that this gap in mathematics achievement may be widening (Campbell, Hombo, & Mazzeo, 2000; Jencks & Phillips, 1998; Lee, 2002; Martin, 2003).

> Mathematics literacy and proficiency has become a fundamental requirement for students

taking advanced mathematics courses, not only in high school, but also at post-secondary institutions. Because of a lack of preparation in mathematics at the secondary level, students are unable to take higher-level mathematics courses at post-secondary institutions. One contributing factor is the number of students entering post-secondary institutions requiring remedial mathematics. In a recent article, Dillon (2009) cited other contributing factors for students taking remedial classes such as poorly ran schools and a lack of connection between high school and

colleges. In an NCES (2003) study, 97% of all public 2-year institutions offered remedial mathematics courses. This statistic compared to other types of post-secondary institutions, which offered remedial mathematics courses such as, 78% of public 4-year institutions and 49% of private 4-year institutions. While these statistics have improved in recent years, there are still a large proportion of students who continue to take remedial courses. Dillon (2009) reported that although precise accounting was somewhat flawed, more than 60 percent of students enrolling at two-year colleges, and 20 percent to 30 percent at four-year colleges, took remedial courses.

Despite the limitations in mathematics access for many Black students, mathematics literacy has and will continue to be a necessary skill for obtaining opportunities beyond high school that are both academic and career oriented. Obtaining a basic level of mathematics literacy has even become a greater predictor of income today than it was decades ago (Carpenter & Bottoms, 2003). As the work towards improving mathematics literacy for Black students, and

all students for that matter continues, some scholars have suggested another way students can engage in mathematics; establishing a community where students are excited about doing mathematics and where students can take a more active role in the teaching and learning of mathematics with their peers.

Research Questions

The purpose of this study was to examine how the experiences of African American college mathematics literacy workers (CMLWs) and high school mathematics literacy workers (MLWs), within a community of practice namely the Young People's Project (YPP) Chicago, influence the identities they have and inform how they categorize numbers used in the Flagway Game. I also examined what goals CMLWs and MLWs had when working in their communities

and the value that they saw in carrying out this kind of mathematics literacy work. This study was guided by the following research questions:

1. What identities do African American college mathematics literacy workers and high school mathematics literacy workers have of the Young People's Project Chicago, in the context of the Flagway Game training?
    a. What led African American college mathematics literacy workers and high school mathematics literacy workers to participate in the Young People's

Project Chicago?

b. What role do African American college mathematics literacy workers and high school mathematics literacy workers see themselves having in their local communities?

2. What are the mathematics strategies used by African American college mathematics literacy workers and high school mathematics literacy workers in number categorization of the Flagway Game and their understanding of these number concepts?

Rationale for the Study

My goal in conducting this study was to investigate CMLWs and MLWs of the YPP Chicago, a community of practice, within the context of their mathematics literacy workshop training. I wanted to understand what aspects of their identities were shaped as they engaged in the training and through reflection of their past and present work implementing a mathematical game, called the Flagway Game.

Despite the widespread awareness of the Algebra Project and the YPP among mathematics educators, there are only a few studies that have examined various aspects of their efforts. These studies have focused on the historical development of one research site in Chicago (N. Cobb, 1994), AP's community development initiatives (Sanders, 1995), an African American program for boys (Nelson, 1997), AP's impact on student achievement in mathematics (Adair, 1996), the impact and effectiveness of the AP's curriculum in an urban high school in Jackson,

Mississippi (Davis & West, 2000, 2004a, 2004b), and how members of a community define and address inequities in mathematics classrooms (Davis, West, Greeno, Gresalfi, & Martin, 2007).

Furthermore, there is little research on the work of YPP as an organizing tool for mathematics literacy and for young people learning mathematics. Educators have documented the development of YPP in Jackson, Mississippi (R. P. Moses & Cobb, 2001), and addressed the importance of their work from a social justice perspective at a Chicago outreach site (Gutstein, González, & Masionette, in press). However, YPP remains an underresearched area in mathematics education offering a fruitful area for inquiry. YPP contributes new knowledge to mathematics education in a variety of ways. Studying YPP reveals the complexity in mathematical content of the Flagway Game and the activities that young people engage in. Studying YPP also offers new perspectives on communities of practice that young people engage in and the commitment they make to community outreach work. Finally, YPP sheds light on the complexities of identity and the changing role that young people take on within these social contexts. *Why choose CMLWs and MLWs of the YPP Chicago?* I decided to focus on CMLWs and MLWs of the YPP Chicago because the organization met two major criteria of this study. First, out of all of the YPP national sites, this one had the most developed outreach program. Second, I wanted the opportunity to work with a diverse student population and select participants from a large sample of CMLWs and MLWs. The YPP Chicago offered me this opportunity, not to

mention the chance to visit multiple sites where trainings occurred.

Significance of the Study

Current mathematics education reform efforts, mathematics educators, policymakers, curriculum developers, and mathematics teachers are investigating new approaches for

improving student learning through professional development and other innovative teaching strategies. Mathematics reform efforts have looked broadly at change with the focus on teachers and the way they teach specific mathematics curriculum. Studies have addressed teachers' belief and how they can change their practice in classrooms (Senger, 1999). Studies have also looked at the effects of reform-based instruction on student outcomes (Baxter, Woodward, & Olson, 2001; Schoenfeld, 2002, Thompson & Senk, 2001; Van Haneghan, Pruet, & Bamberger, 2004). Rarely if ever are youth-led initiatives included as part of the reform conversation (Wilson & Corbett, 2007) in general, and as a strategy for change in schools to improve mathematics literacy in particular.

This study contributes significantly to mathematics education in three unique ways. First, YPP is the first program of its kind conducting youth development and youth-led mathematics initiatives in underserved communities with the goal of systemic change in mathematics education. Second, this study takes a look at CMLWs and MLWs within a community of

practice of young people and what identities they have within this context. Studying communities of practice is an emerging area of inquiry in mathematics education. Finally, this study contributes to the existing research by giving voice to CMLWs and MLWs who come to mathematics literacy work with a primary objective of creating change in their community by helping children learn mathematics in a unique way. Giving voice provides CMLWs and MLWs with the opportunity to speak for themselves about their experiences rather than being spoken for by others. Studying a youth-led initiative of YPP serves as an exemplar of what happens when mathematics literacy workers are able to take ownership of mathematics with the purpose of improving their own understanding as well as the mathematical experiences of others. My main focus in exploring the above research questions was to understand how CMLWs and MLWs see

themselves and the understandings of numbers they extrapolate from their use of the Flagway Game. I also wanted to study how the YPP experience contributes to the identities they formed.

Limitations to the Study

This study is limited to the CMLWs and MLWs of YPP Chicago at one Flagway Game

Workshop training. Researching one workshop training limits the ability to generalize across various sites in Chicago and at other YPP sites throughout the country. Because activities at each training site were unique, there may be other factors that influenced identity at other sites in

Chicago and at other YPP sites around the country not addressed in this study. I sought to conduct an in depth study on identity and mathematical understandings of CMLWs and MLWs; so the number of participants used in this study is also a limitation. Although a larger sample size may have provided more generalizable results, the small sample size allowed me to gain an in depth perspective on what identities CMLW and MLWs had and how their understanding of numbers in Flagway was enhanced. Because of participant characteristics, this study is also limited to their age and experience in YPP.

Definition of Terms

Throughout this report, several acronyms and terms are used repeatedly. For the purpose of clarity, these acronyms and key terms are defined as follows:

- *Abelin Preparatory High School* – Pseudonym; location of MLW Flagway workshop training and high school attended by two of the participant
- *Agent of Change* – Someone who purposely works toward creating some kind of social, cultural, or behavioral change in society or in others through his or her work or actions
- *Algebra Form* – An algebraic representation, with variables, used to symbolize or stand in for numbers used in a mathematical expression

- *AP* – the Algebra Project
- *CMLWs* – College Mathematics Literacy Workers

• *CMLW Flagway Game Workshop Training* – Facilitated by YPP instructors, CMLWs learned how to facilitate workshop training, play the Flagway Game, and use mathematics activities with MLWs

• *Communities of Practice (CoPs)* – A community of practice is a collective group, unified by common interests where members interact regularly in order to create and improve what they learn and share over time

• *Composite Number* – A whole number greater than 1 that has more than two factors

• *Confident Doer of Mathematics* – An individual who came to YPP confident in his or her ability to do mathematics

• *Even Number* – Any integer that can be divided by 2 without leaving a remainder

• *Factor* – Any of two or more numbers multiplied together to form a product

• *Factor Tree* – A tree-like diagram used to find the prime factorization of a positive integer greater than 1

• *Identity* – Viewed from a situated perspective, it is a negotiation between learners and the various social contexts in which they participate in that influence how they view themselves

• *Increasingly Confident Doer of Mathematics* – An individual who demonstrates a sense

of confidence or willingness to do mathematics that developed during the project

- *Mathematics Literacy Workers* – College and high school students in YPP that did mathematics literacy work
- *MLWs* – High School Mathematics Literacy Workers

- *MLW Flagway Game Workshop Training* – Facilitated by a YPP instructor and CMLWs, MLWs learned how to facilitate workshop trainings, play the Flagway Game, and use mathematics activities with elementary students
- *Multiple* – A multiple of an integer is the product of that integer and another integer
- *New University* – Pseudonym; university attended by two of the study's participants
- *Number Sense* – The ability to work with and understand numbers and their relationships, flexible mental computation, quantitative judgments, and how numbers are affected by mathematical operations
- *Odd Number* – Any integer that cannot be divided evenly by 2
- *One Who Preserves* – Someone who continues to persist in or remain constant to a purpose, idea, or task in the face of obstacles or discouragement
- *Outreach Sites* – Elementary schools where CMLWs and MLWs engaged children in mathematics activities to support them learning and playing the Flagway Game
- *Prime Factors* – Positive prime integers that divide into that integer evenly, without

leaving a
remainder

- *Prime Number* – A whole number greater than 1 that has only two factors: 1 and itself
- *Role Model* – A person who serves as an exemplar of positive behavior in one or more

  context
  s
- *Student Voice* – Giving students a legitimate opportunity to explicitly address their

  concerns about schools and play an active role in changing how schools function
- *Supporter of Others* is defined to be a person who contributes to the fulfillment of a need

  or furtherance of an effort or purpose.

- *Training Hubs* – The six sites in Chicago where Flagway Game workshop training took

  place
- *Trajectory* – A continuous motion of coherence through time that connects the past, the

  present, and the future; critical in identity formation since renegotiation occurs

  throughout the course of an individual's life (Wenger, 1998)
- *YPP* – the Young People's Project
- *YPP Instructor* – Former CMLWs that assisted in conducting CMLW and MLW training

  and supported mathematics literacy workers

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CHAPTER 2

REVIEW OF LITERATURE

Just because you don't see change the way you want it doesn't mean that a change didn't happen. Or just because you are not affecting community the way you think community should be affected doesn't mean that an [effect] didn't happen. (Naomi, College Mathematics Literacy Worker, 2007)

When young people are actively engaged in activities that affect change through forms of mathematical teaching and learning, this engagement enables their own learning and strengthens their identities. This review of literature is organized into five sections: (1) background on the Algebra Project and the Young People's Project, (2) student involvement in school change, (3) critical race theory and education and (4) communities of practice, learning, identity, and mathematics in a social context and (5) research on functions and decompositions. The first section provides a summary of both the AP and the YPP. In this section, I also include the importance of Chicago as a race-place context. The second section reviews the emerging literature on student involvement in school change, and includes a brief historical overview. I discuss the importance of including student voice in school change and research that supports this inclusion is then summarized. The third section provides an overview of critical race theory (CRT) and how its relevance in the field of education. I also include studies that have used the CRT in giving voice to contest race and achievement of Black people. The fourth section I provide a discussion on communities of practice, with emphasis given to the modes of belonging and the processes of identification and negotiability in identity formation. I review literature on mathematics, learning, and identity within a social context. I also provide the usefulness in discussing mathematics and identity in the social contexts of learning. Finally, in the fifth section, I include literature that discusses students' understanding of functions and decomposition

.

The Algebra Project and the Young People's Project

The Algebra Project

The Algebra Project (AP) is a national initiative, rooted in the U.S. Civil Rights Movement that is carried out in schools and after-school programs. The main purpose of the AP is to improve the mathematics literacy of young people of color underserved by existing education reform efforts in order for them to gain political and economic power and access to opportunities. As R. P. Moses and Cobb (2001) describe, "The Algebra Project is first and foremost an organizing project – a community organizing project – rather than a traditional program of school reform....Like the civil rights movement, the Algebra Project is a process, not an event" (p. 18). AP's goals are accomplished through mathematics curriculum materials development, teacher training and support, youth mathematics literacy development, and school-community partnerships. For more than two decades, AP has received national attention for its mathematics reform efforts in inner city and rural schools (R. P. Moses, 1994; R. P. Moses & Cobb, 2001; R. P. Moses, Kamii, Swap, & Howard, 1989; Silva, Moses, Rivers, & Johnson, 1990).

Founded in 1982 in Cambridge, Massachusetts, by civil rights activist Robert Moses, AP has two main goals. The first is "to increase the proportion of students who complete algebra successfully in late middle school or high school and enter college preparatory studies" (Davis &

West, 2000, p. 1). The second goal of AP is the development of mathematics literacy among disenfranchised students in urban and rural communities. In response to issues surrounding how algebra is taught to Black students of low socioeconomic status (SES), AP works to develop mathematics proficiency through an experience-based approach to learning and pedagogy.

AP's curriculum combines inquiry and experiential learning, which is mathematics emerging from human experience. Mathematics is also made accessible by using real-life situations, like a bus or train ride, and African drumming because they embody rich mathematical concepts. Through the process of mathematizing these situations or events, students are encouraged to actively engage in mathematical discourse by using their everyday language for talking about mathematical concepts. This discourse leads to a focus on important mathematical features about the event and to the process of symbolization. By actively engaging in the mathematics discovery process, students encounter complex mathematical ideas that they learn to work through.

As in the civil rights era, when voting was identified as a requirement for citizenship, AP has identified new criteria for citizenship in today's society: mathematics literacy, but more important, accessing algebra. Our society has grown increasingly competitive and complex, and

students residing in poor, marginalized communities are confronted with the challenge of meeting educational demands at an especially earlier age. Consequently, the ability of students in these communities to develop mathematics literacy and proficiency in algebra and strengthen their prerequisite knowledge for algebra becomes even more imperative. Although the AP started to address the importance of algebra as early as the mid-1980s, the results of national studies such as the Third International Mathematics and Science Study (Mullis, Martin, Beaton, Gonzalez, Kelly, & Smith, 1998), underscore the need for U.S. schools to strengthen their secondary curriculum and build the foundation of mathematics literacy for all students in earlier grades.

Algebra has long been seen as a gatekeeper (Chappell, 1997) to higher-level mathematics courses and opportunities. Adequately preparing marginalized students to pass through the gates has become the main mission of AP's work. Chappell argues:

> Preparing students to enter the gate to algebraic thinking contributes to minimizing the differences in mathematics-course participation and achievement that have long existed between males and females and different racial and ethnic groups. To close these gaps in achievement, we need to open the gate to algebraic thinking. (p. 267)

In light of this increased demand for mathematics literacy and proficiency, AP works in urban and rural communities to make algebraic ideas more contextual, meaningful, and realistic.

Like the National Council of Teachers of Mathematics (NCTM, 2000) *Principles and Standards for School Mathematics*, AP promotes constructivist (P. Cobb, 1994; Clements, 1997; Clements & Battista, 1990; von Glaserfeld, 1983) ideas for student learning and nontraditional approaches to teaching. In *Radical Equations*, R. P. Moses and Cobb (2001) describe how they use the constructivist approach in the AP curriculum to provide students with everyday mathematical experiences.

> In the late 1980s, the curriculum development work of AP targeted students in middle

school, where many of them were exposed to algebraic concepts and thinking for the first time. This was called the Algebra Project Transition Curriculum. AP was awarded the Instruction Materials Award by the National Science Foundation (NSF) to build partnerships with research mathematicians. The NSF award enabled Bob Moses to make the curricular shift to develop materials for secondary mathematics classrooms. The award also provided AP with credibility in the mathematics community. The Transition Curriculum was not used in schools after the early 1990s due to the increased demand for high stakes testing.

In addition to offering the traditional algebra course, some middle and high schools also adopted the AP curriculum as an alternative way of introducing algebra to underserved student

populations. Students voluntary opted out of traditional algebra classes and enrolled in an AP class with an agreement to take the course 5 days a week for an hour and a half during a block

schedule. AP-trained teachers facilitated the AP classes for the entire school year. On completion of a year of algebra with AP, students entered the next level of mathematics at their respective schools. At only one site, Lanier High School, have AP students gone through an entire 4-year mathematics sequence using AP-developed curriculum materials. Other schools have yet to adopt this format. Since its inception, AP's curriculum has reached over 40,000 students in 13 states, including California, Massachusetts, Mississippi, Illinois, and New York, as well as 28 school districts (Algebra Project, 2006).

The AP's grassroots approach to educational change positions it to play a significant role in education reform in general and mathematics reform in particular. Robert Moses and his work in the AP have also helped to pioneer a new grassroots effort, the Quality Education as a Constitutional Right (QECR) movement. The QECR movement began in 2005 at Howard University, where scholars, activists, parents, students, and other community stakeholders came together to discuss "how to ensure that a quality education is guaranteed to all [of] America's children" (QECR.org, 2008, QECR: History). The primary goal of QECR is to build consensus

with political leaders and community stakeholders in developing a constitutional amendment guaranteeing a quality education as a *civil right* for all children so that they can gain access to full citizenship in today's society.

AP continues to work in rural and urban communities. The youth-affiliated organization, the Young People's Project, also works in these communities to improve the mathematics

outcomes of students through community outreach efforts.

The Young People's Project

The Young People's Project (YPP), is a youth-led organization that is an outgrowth of, and in partnership with the Algebra Project. The YPP was founded in Jackson, Mississippi, in 1996. YPP is a youth-empowerment and after-school mathematics initiative created by an alliance of Black students who were AP graduates of Sam M. Brinkley Middle School in Jackson, Mississippi. Brinkley AP graduates wanted to take an active role in their community by preparing youths to become more mathematically proficient. YPP was "founded on the belief that there is work that young people can and must do to change the conditions of their lives and that math literacy work was a good place to start" (Tyyp.org, 2009, YPP Math Literacy + Social Change: History). They also believed this work was necessary, because Mississippi remains one of the most economically depressed and poorest performing states in the nation when it comes to education. Performance of Black schools also remains at the bottom in the state (Southern Education Foundation, 2006). According to National Assessment of Educational Progress (NAEP) data, Mississippi students are far behind students in most other states in every grade and subject area. Specifically in mathematics, fourth and eighth grade students in the state scored at least one grade lower than the average student in the nation (NCES, 2005b). Quoted in Robert Moses' *Radical Equations,* Mae Bertha Carter, a civil rights activist, talked about the failure of

Mississippi schools to adequately prepare children:

> The way to control Black people or anybody is to keep them dumb. You keep them dumb and you can control them. Back in slave time they catch you reading and they would whip you. Education, that's the goal. Getting the knowledge and understanding. If you are uneducated you don't know nothing. You don't know what's going on around you. So what they're doin' is handicapping kids. These school systems ain't doing nothing but handicapping these children. (Moses & Cobb, 2001, p. 134-5)

YPP has three main objectives. The first is to use mathematics literacy to develop youth leaders and organizers who would radically change the quality of education and life in their communities (O. Moses, 2006). The second objective is to develop young people as facilitators,

mentors, and advocates for mathematics literacy. The third objective is to assist a target population of AP students and non-AP students to successfully complete algebra by eighth and ninth grade and to enter a college preparatory mathematics sequence in high school. YPP works to achieve these objectives through the development and implementation of mathematics games and activities in schools. Addressing these objectives is different from city to city because YPP's scope of activities varies across sites. The work of YPP is tailored to meet the demands of the community it serves.

Members of both AP and YPP believe that young people in today's society must play an active role in helping students achieve mathematics literacy (R. P. Moses & Cobb, 2001). Moses also believes that young people need to be inspired to fight for their own liberation. Like the AP, which is guided by the belief that all children, regardless of their racial, cultural, or

socioeconomic background, can learn algebra as well as other areas of mathematics, YPP develops and prepares college and high school students, who are known as college mathematics literacy workers (CMLWs) and high school mathematics literacy workers (MLWs), respectively, to market their mathematics skills in both after-school programs and through an AP network composed of activists, educators, parents, and teachers. CMLWs and MLWs have presented their work at regional conferences of the NCTM and at the Mathematics Science Research Institute in Berkeley, California, which has given them the opportunity to demonstrate the materials they use in their community work to the broader mathematics and mathematics education community.

After successful preparation in training workshops by YPP instructors, CMLWs and MLWs lead their own training programs for new students, which are coordinated weekly in schools. CMLWs and MLWs organize homework help centers, training sessions, and summer institutes. At several YPP sites throughout the country, supplemental mathematics curriculum

materials are developed and used in AP classrooms and in workshops where the mathematics skills of CMLWs and MLWs are demonstrated to peers, mathematics teachers, and parents.

Omowale Moses, son of Robert Moses, a founding member, and the executive director of YPP, further explained the mission of the organization:

The goal of the YPP training process is to enable MLWs to develop math skills in a way

> that builds a culture of youth leadership, ownership, and teamwork. We also attempt to produce young people who can effectively teach, mentor and tutor other students. (Typp.org, 2008, About the Young People's Project)

The organizing initiative of the YPP follows the legacy of the Student Non-Violent Coordinating Committee (SNCC), a grassroots student-led organization during the civil rights movement (Bond, 2000; Carson, 1981). At the forefront of the civil rights movement were young people in SNCC who played an integral role in changing the racial, economic, and educational conditions in their communities. College and high school students organized voter registration drives, marches, and sit-ins, demanding to local, state, and national officials that people of color deserved voting rights, access to opportunities, and a legitimate place in society. YPP continues this tradition by organizing young people to demand their right to a quality mathematics education not only through public demonstrations (Schoettler, 2004), but by working with students to achieve the accessibility of mathematics in underserved communities.

The YPP was created by young people for young people. In the spirit of youth leadership, YPP serves as a tool for inspiring students to play an active role in the mathematics education of others while empowering their peers. Similar to the way that sharecroppers fought for their own voting rights in the 1960s, YPP gives students of color ownership of learning mathematics. Furthermore, YPP enables students to make the argument on their own terms, through their outreach work, that mathematics is important in today's society. The statement that follows summarizes how the YPP hopes to shift how young people begin to see themselves:

> We want young people to harness the power of their culture to work for them and not against them. We want young people to challenge the dominant cultural logics that help to marginalize them. We want young people to reject any cultural expectations that limit who they can and should want to be. YPP wants to make it cool for young people to learn and share what they know.... Young people need to broaden their cultural expectations about what's cool, what's acceptable, and what's respectable -- if we can celebrate each other for being young athletes and hip hop artists we can celebrate each other for being young scholars, organizers, and orators. (Young People's Project, 2008, p. 3)

Over the years, YPP has expanded to several cities, including Greater Boston and Chicago, and is developing sites in Miami, FL; Detroit, MI; New Orleans, LA; Oakland, CA; Rochester, NY; Petersburg, VA; and Yuma, AZ. For my study, I selected YPP Chicago.

YPP Chicago. YPP Chicago was started in the summer of 2002 through a partnership with the Chicago Public Schools Mathematics and Science Initiative (CMSI) and After-School Matters (ASM). CMSI is a comprehensive strategic plan to improve mathematics and science instruction in all of Chicago public schools. ASM is a Chicago-based non-profit organization that offers out-of-school opportunities to city teens. Maintaining an office at ASM headquarters, YPP partners with the two programs to operate training hubs throughout the city of Chicago for the development of CMLWs and
MLWs.

Each of the training hubs is affiliated with one of three local institutions of higher learning: DePaul University, the Illinois Institute of Technology, and the University of Illinois at Chicago. The goal at each training hub is to develop and prepare a team of CMLWs and
MLWs

to conduct training programs in Chicago public schools, churches and community organizations for middle and elementary students, parents, and community members. There are six training hubs in Chicago: four high schools and two universities. YPP Chicago has prepared over 200 high school and 50 college students to conduct math literacy workshops and community events. CMLWs and MLWs carry out math literacy workshops in 20 sites in the North, South, and Westside communities of Chicago (O. Moses, 2006).

YPP Chicago seeks to develop mathematics literacy in algebra for eighth and ninth graders and develop number sense with elementary students in Grades 3 to 6 through the Flagway Game program. The program engages students in mathematics numeracy through mathematically rich games and activities. This approach is taken to "radically change how and what students learn about their first 150 numbers" (O. Moses, 2006, p. 33). Furthermore, because of the competitive aspect of the Flagway Game it "seeks to create an opportunity for students in Chicago to learn and celebrate learning math, in the same way that they learn and celebrate learning basketball" (p. 26). Further, although there are several state-funded mathematics initiatives in Chicago, many are book and worksheet focused, geared toward students taking and passing the Illinois State Achievement Test (ISAT). Consequently, Chicago youths are not afforded many opportunities to learn mathematics outside of a school context like YPP.

The Race-Place Context, Chicago, and Black Educational Disenfranchisement

Scholars have highlighted the importance of foregrounding the interconnectedness of *race and place* in recent literature (Bullard, 2007; Frazier, Florence, & Eugene, 2003; Morris & Monroe, 2009). Place should not be separated from an analysis of race in the United States because each place is critical (Morris & Monroe, 2009) and comes with unique issues. Scholars argue that in order to fully conceptualize and understand educational achievement in general, and a Black educational experience in particular, the context of place and the cultural, historical, racial, political, and economic distinctions that each brings must always be considered to gain a deeper and more holistic perspective (Morris & Monroe, 2009). In my effort to set the stage, in this section, for a brief yet broader discussion of geographical location and its influence on African American schooling and achievement, I consider Chicago, as a race-place context for YPP Chicago.

Chicago is inextricably linked with the South in many ways. Chicago was the site of a major population shift of Blacks from southern states like Mississippi as early as the First World War. Black migrants flocked to the urban North in search of better jobs and a better way of life. The exodus from the South is known as the Great Black Migration, and it began as early as 1910, lasting until the early 1970s (Drake & Cayton, 1962). Black migrants arrived to Chicago from southern states hopeful of a new life and what the big city could offer their children with regard to education. However, many were met with racism and discrimination from Whites,

which

paralleled Jim Crow laws in the South. Many Blacks were systematically excluded from exercising their civil rights when trying to gain access to basic life necessities like public accommodations and services, jobs, adequate housing, and quality schooling for their children. Furthermore, in Chicago, Whites demarcated strict color lines, which bounded where Blacks could and could not reside and go to school. Consequently, Blacks were restricted to densely populated districts known as enclaves. Some enclaves later became slums and ghetto neighborhoods that were difficult to relocate out of (Spear, 1967). Because of systemic racial discrimination and economic deprivation, many Blacks in Chicago experienced continuous disenfranchisement. These events, among others, laid the foundation for a biracial Chicago, a White one and a Black one. These two Chicagos still exist today.

Although Chicago has been a major urban center of media, business, and political opportunity for Blacks, it fares poorly in comparison to other major cities when it comes to educational achievement of students, particularly in mathematics. According to a recent report using TIMSS assessment data, the Chicago Public Schools performed significantly lower on mathematics achievement tests as compared to other school districts in Illinois and other states around the country (Wright et al., 2003). This distinction is important since the Chicago Public

Schools (CPS) is the largest school district in the state and third largest school district in the

nation. CPS is composed elementary schools, high schools, and charter schools (e.g., elementary and high school). The racial breakdown of students attending CPS was approximately 46.2% of African Americans, 41.2% of Latinos, 8.9% of Whites, and 6.6% of other ethnic groups. In 2007, the Chicago Public School graduation rate was approximately 55.1% citywide, with only 50.1% of Black students graduating. Moreover, approximately 84.3% percent of all students attending CPS were from low-income households (CPS, 2009). Furthermore, Chicago had the sixth highest Black poverty rate among 23 major cities around the nation.

The Chicago area ranks fourth in the nation in African American and White school segregation. Segregation is seen mostly in urban communities where large concentrations of Black people reside and attend schools. Massey and Denton (1993) define segregation as "the general tendency of blacks and whites to live apart" (p. 74). They conceptualize segregation quantitatively along five dimensions: unevenness, isolation, clustering, concentration, and centralization (Massey & Denton, 1988). They further assert that a metropolitan city is considered to be hypersegregated if it is "very highly segregated on at least four of the five dimensions at once" (Massey & Denton, 1993, p. 74). In a study on U.S. metropolitan areas with large Black populations, according to their estimates, Chicago was identified as a hypersegregated city along all five dimensions.

African Americans continue to be the most segregated racial/ethnic group in the United States (Bullard, 2007). The issue of segregation or hypersegregation becomes a critical issue when considering its connection to school funding (Carey, 2003). Illinois ranks forty-ninth

nationally in the amount of educational funding provided by the state for each student. Funding disparity ranges from $18,225 per student in predominantly White areas of the state to $6,678

per student in areas like Chicago, where predominantly non-White students attend school (Center for Urban Research and Learning, 2006). These differences in spending per student reflect explicit racial education gaps. Discrepancies in funding affect the types of educational resources that are available to all students, educational quality, and achievement. These discrepancies provide some explanation for the racial inequities that continue to exist in Chicago schools today. When all students are not afforded equitable resources for a quality education, that limits how and where they participate in society. Foregrounding Chicago as a race-place context provides impetus for discussing YPP in Chicago. The emergence and work of YPP Chicago is a response to and an effort towards combating some of the inequities that exist among Black disenfranchised youth.

Student Involvement in School Change

Students' Historical Involvement in School Change

Encouraging students to play an active role in their education and school reform movements is not an uncommon phenomenon. Historically, students have exercised their right to

participate in the decision-making process of what happens in schools and in classrooms (Cusick, 1973; deCharms & Roth, 1976). Inclusion and participation were a legacy of civil rights movements and evident in student power movements of the 1960s and 1970s. During that time, the involvement of students in school change efforts was largely around the issue of democracy and political involvement. Students actively questioned the status quo and carefully worked to transform the educational and political conditions in schools, universities, and their communities (Carson, 1981; Williamson, 1999). Since that time, the role of students in the decision-making process of school change has diminished considerably. Students' voices have become even more silent in schools, where young people experience alienation, tracking by ability and age, and

students have a conception of themselves as clients as opposed to learners. This has created a distance between students and teachers (Costello, Toles, Speilberger, & Wynn, 2000; Mitra, 2004; Nieto, 1994; Pittman & Wright, 1991; Soo Hoo, 1993).

Levin (2000) explains that in the mid 1970s there was a shift in the idea of student involvement in school decision-making to students taking a more passive role in schools. Students have become the "objects" of change, "objects to be worked upon rather than actors to be taken seriously" (p. 164). Fullan (1991) also gives a summary of how students are now viewed and its relation to school change. He writes:

When adults do think of students, they think of them as the potential beneficiaries of

> change. They think of achievement results, skills, attitudes, and jobs. They rarely think of students as participants in a process of change and organizational life. (p. 170)

More recently, the discussion of students' involvement in school change is resurfacing in education literature as a necessary requirement to any reform effort's effectiveness (Fielding, 2001). Even with the necessity and recent attention to improve student performance and outcomes in areas like mathematics, students are still rarely given a role in school reform in the United States, "despite the fact that many reforms are intended to create more equitable and engaging educational programs for students" (Mitra, 2004, p. 652).

Although the literature in this area was quite limited, I decided to include theoretical arguments and studies that spoke to the importance of student voice in the school change process along with studies that focused on various aspects of student participation in actualizing change. In doing so, my objective was to understand how student voice has been *actualized* in schools and what aspects of student participation have been emphasized in the literature. I also wanted to review literature that focused on how student voice has been "conceptualized" and perceived.

Importance of student voice in school change. When students are allowed to voice their opinions about what happens in schools, it gives them the opportunity to share their views about problems and come up with real solutions. More importantly, when students who are disengaged in the educational process are asked their opinions about school improvement, they are given a

sense of purpose and promise (Nicholls & Thorkildsen, 1995). There is mutual benefit to both teachers and students when this kind of work is done. As Erickson and Schultz (1992) suggest, "The absence of student experience from educational discourse limits the insight of educators as well as that of students" (p. 160).

Lodge (2005) urges a critical examination of what is meant by student voices and involvement. When we decide to speak for some, we are unintentionally leaving out others. Some scholars also argue that if students play a more active role in shaping what school reform looks like, that will greatly improve student outcomes and reform efforts (Levin, 2000; Mitra, 2004). Johnston and Nicholls (1995) point out that teachers often come into contact with students who try to develop their own authority and are yearning for their voices to be heard in schools. Recent literature suggests that giving students authority by including them in school change efforts can serve as a vehicle for improving teaching, curriculum, teacher-student relationships, student assessment, and teacher training (Fielding, 2001; Mitra, 2003; Oldfather, 1995; Rudduck & Flutter, 2000; Soo Hoo, 1993). It has also been claimed that improved behavior, better staff-student relationships, higher attendance, and higher levels of achievement have all been attributed to students' inclusion in the decision-making process in schools (Biermann, 2005). Nieto (1994) also underscores the importance of student voice by noting its scarcity in school policies and practices. She writes:

> One way to begin the process of changing school policies and practices is to listen to students' views about them; however, research that focuses on student voices is relatively

> recent and scarce. For example, student perspectives are for the most part missing in discussions concerning strategies for confronting educational problems. In addition, the voices of students are rarely heard in the debates about school failure and success, and the perspectives of students from disempowered and dominated communities are even more invisible. (p. 396)

To help students and other stakeholders find a meaningful role in school reform, Goldman and Newman (1998) offer strategies on how to get them started in improving schools. Students, in particular, are able to define reform needs as well as the opportunity to put many of them into practice. Furthermore, diverse student populations are included in school processes.

Scholars interested in the issue of student voice and student involvement have asked, "What role does student voice have in the decision making process of school change or reform?" In response to this question, Lincoln (1995) suggests that the role of student voice is embodied within three distinct contexts in which we place students: a social and legal context, a scientific context, and a political context. In the social and legal context, student voice is a necessity that reflects the long history of civil rights efforts in this country. Unlike the past, where the rights of children were not considered, today's orientation brings awareness to many that children are indeed citizens and benefactors of the future and that their ideas should be heard and respected. In the scientific context, attention to student voice brings an understanding of how learning occurs. Schools are spaces where students make sense of their world and some of the most influential sites for learning. In this way, attention to how students actively formulate their

realities in the social world offers new promises and opportunities for learning. Students are also placed in a political context as well. Lincoln argues that the purpose of education in this country has been for democratic participation even though we have not achieved that purpose effectively. The role in student voice is to support a "democratic, just, and economically viable and

prosperous society [that] requires active participation and critical thinking skills far beyond what many of our students experience in school" (p. 89).

Corbett and Wilson (1995) took a slightly different approach in their discussion of the importance of students in the changing process of reform. They argued that student participation is not independent of reform. We must consider the role of students in the reform process, not separate from it. If we have the expectation that students should change, then our expectations of them should change as well.

Although there is a growing body of research that argues for the merits of student voice, there are very few studies that have actually examined how student voice is used to create change

in schools. Although some of these studies have been conducted in the United States, many have been conducted in other countries, like Australia, Canada, and the United Kingdom. The focus of these studies has been on rights and empowerment of students as in the past, particularly during the U.S. Civil Rights era. But current attention is on the effects of student voice and involvement

on student outcome and school reform, with a particular emphasis on the various ways student voice was sought (Mitra, 2004).

Research on Student Involvement in School Change

Pupil participation and school change. Rudduck and Flutter (2000) argue that pupil participation and perspective are important aspects of changing curriculum and instruction. The relevance of curriculum is usually an adult view of what is important rather than a pupil view of what young people find meaningful. In interviews conducted with pupils in primary and secondary schools in the United Kingdom, they found that pupils were interested in "changing the structures that cast them in a marginal role and limit their agency" (p. 84). Rudduck and

Flutter's interviews allowed them to construct a model of pupils' commitment to learning and their identity as learners. They argue:

> The regimes of school, which embody values operating through structures and relationships, shape pupils' attitudes to learning and their view of themselves as learners. The more that the regimes are changed to reflect the values that pupils call for (intellectual challenge, fairness, etc.), the stronger pupils' commitment to learning in school is likely to be. (p. 85)

The model that Rudduck and Flutter (2000) offers is interesting because it shows what structures need to be in place or what structures need to "change" within a certain context to realize the pupils' commitment to learning. For example, certain structures that support student participation

as opposed to others.

Schratz and Blossing (2005) provide their individual perspectives on pupil participation in school change in a two-part article. Schratz argues that pupils should be able to make decisions about school change because it directly affects them. Moreover, pupils are capable because they are required to make decisions about other important aspects of their lives every day. Schratz believes that in order for students to be effective in this kind of work, they should be equipped with the right literacy. Literacy is described as the everyday decisions young people are required to make to help them learn about life and their competency in doing so. He also argues that the power structure in most schools does not promote pupil participation. In order for student involvement to be effective and taken seriously, our view of teaching and learning must shift.

Blossing holds similar views to those of Schratz and contends that pupil participation is a key component to ensure any implementation effort in schools. Blossing argues that pupil involvement is a matter of democracy because an important aspect of a decision process is organizing "various views of interest" within a dialogue.

Levin (2000) offers a pragmatic line of reasoning for greater student participation that is based on five factors:

1. Effective implementation of change requires participation buy-in from all those involved, students no less than teachers; 2. Students have unique knowledge and

> perspectives that can make reform efforts more successful and improve their implementation; 3. Students' views can help mobilize staff and parent opinion in favour of meaningful reform; 4. Constructivist learning, which is increasingly important to high standards, requires a more active student role in schooling; 5. Students are the producers of school outcomes, so their involvement is fundamental to all improvement. (pp. 156-157)

Student voice in school change. In the United Kingdom, a 3-year study project, Students as Researchers, was conducted at Sharnbrook Upper School, where students adopted various roles throughout the project to address specific issues within their high school (Fielding, 2001). In Year 1 of the study, a student cohort of mixed age and gender took on the researcher's role to identify important issues and problems that needed to be addressed from daily interactions with teachers and students. Student researchers were then asked to make recommendations to be shared with their peers, staff, and governing bodies in the school. In Year 2, Students as Researchers continued in the same way as the year before, but others within the cohort became student consultants and offered support and their expertise to a new cohort. Year 3 was an extension of the work from the prior year, and the methods of research done during the Students as Researchers project were shared with other schools in the United Kingdom as well as abroad. Fielding reported that to both acknowledge and promote student voice, the degree to which these key components can be demonstrated by students was important. A framework to honor their input had to be in place. The framework considered the following topics: speaking, listening, skills, attitudes and dispositions, systems, organizational culture, spaces, action, and the future.

Students as researchers is discussed elsewhere in the literature as a relatively new approach to research, and there are only a handful of studies that have engaged in this method of inquiry (Alder & Sandor, 1990; Atweh & Burton, 1995; Boomer, 1987; Knight, 1982; Schwartz, 1998; Slee, 1988). In every one of these studies, the main goal was to have students experience the process of research. In addition to the work that Fielding (2001) reported, only two of the students as researchers projects were close to a focus on educational change (Atweh & Burton, 1995; Slee, 1988). Slee conducted a study to identify the educational needs of 13-14 year olds not met by the educational system. Atweh and Burton also conducted a sociological study. It focused on the factors that affect students' choice to remain at school. The study was based on the premise that socioeconomic status influences students' participation in schools.

Student voice efforts have also been connected to youth development outcomes. In a study at Whitman High School, located in a northern California community, the goal was to

maximize the opportunities to observe student involvement in the school's reform work (Mitra, 2004). There were two student groups at Whitman that worked in isolation but engaged in student voice activities: Pupil-School Collaborative and Student Forum. The two groups shared similar goals for school improvement by involving students more directly in reform efforts. Student voice efforts included collecting information from students, participating in focus

groups, and administering surveys to students who worked closely with teachers to determine the best strategies for school improvement. The findings from the study showed that for students who were members of either group, their direct involvement in school change efforts led to increases in youth development. Youth development in this case was attributed to three qualities: agency, a sense of belonging, and competence. Students from both groups constructed new roles for themselves as change makers in their schools, offered opportunities for the youth at Whitman

to cultivate a sense of belonging, and critiqued their environment, where injustices were identified. Nieto (1994) conducted a study with students from a variety of ethnicities, developing extensive case studies to understand the benefits of multicultural education for students with diverse backgrounds. Students provided their views on a wide range of topics, including school policy and practice, curriculum, pedagogy, and racial discrimination. In interviews students expressed their concerns about representation of their families and communities in their schools. Students also voiced their concern about teachers maintaining traditional practice, despite the fact that they were undergoing a reform in their schools.

In a study at Seacrest High, another high school in northern California, administrators conducted focus group interviews to determine why failing students believed they were unsuccessful in school (Mitra, 2001). Students were able to give a clear rationale for their

struggles. Mitra found that many of the reasons that students gave for not performing well in school did not correlate with the rationale given by teachers. Giving students the opportunity to voice their concerns helped the teachers to think critically about the assumptions they made about students, and to take reform efforts seriously.

Another relevant study reported that student voice efforts produced a new identity in students as change makers in their school (Mitra, 2004). One way student identity became salient was by having students engage in conversations about reform and providing a student perspective at professional development training sessions. Elsewhere Mitra (2003) suggests that increasing student voice improves student learning, especially when it is directly associated with changing curriculum and instruction. This claim is also substantiated in Rudduck and Flutter's (2000) work on pupil participation.

The literature speaks broadly about student voice and student involvement in school change. Researchers included students who attended schools where change efforts took place. The studies spanned areas that included student voice in teaching, curriculum, and various policies and procedures, as well as student participation in school change efforts. There were also studies that focused on students being a part of the research process, gathering data on behalf of other researchers. These studies were needed because they focused on students involved in change efforts in some way. However, these studies did not focus on the school change efforts of

students in a specific subject, nor did they focus on students making changes to curriculum who were not attending the same school. More important, none of the studies examined students who created intervention programs in schools they did not attend. Finally, I did not find any studies that examined the role that students played in changing what happens in mathematics classes.

Critical Race Theory and Education

Critical Race Theory

Critical race theory (CRT) is an oppositional theory that challenges dominant narratives of widely held hegemonic truths. Through a process of deconstructing why these truths are held, the deployment of CRT creates spaces for new kinds of narratives that have not been considered before. CRT borrows from several theoretical traditions such as law, critical theory, feminist theory, and postmodern theory among others. Critical race theory grew out of legal studies and was first developed in the mid 1970s in response to the failure of critical legal studies (CLS) in addressing race in the legal system. CLS is a well-known movement that explores the way the law conceals culturally accepted norms and standards in society. Crenshaw, Gotanda, Peller, and Thomas (1995) explain that although the early works of CLS were in fact pioneering, the dissatisfaction that many scholars of color had "stemmed from its failure to come to terms with

the particularity of race, and with the specifically racial character of 'social interests' in the

racialized state" (p. xxvi). Legal radicals gave a critique of liberalism but fail to address how race and racism were deeply embedded in American life (Cook, 1995; Matsuda, 1995). CRT grew out of the work of legal scholars, Derrick Bell, Alan Freeman, and Richard Delgado (Delgado & Stefancic, 2001), who examined the historical centrality and complicity of law in its perpetuation of racism and white supremacy. Cornell West (1995) explains that "critical race theorists, for the first time, examined the entire edifice of contemporary legal thought and doctrine from the viewpoint of law's role in the construction and maintenance of social domination and subordination" (p. xi). In West's view, scholars of color deconstructed the basic assumptions of mainstream liberal and conservative ideology in the legal academy.

In addition to the critique of race in the legal system, CRT also embodies an emancipatory agenda as well. Scholars across disciplines have combined CRT with other critical epistemologies in order to argue for a social justice agenda, which puts issues of race and other forms of discrimination in public discourse (Parker & Lynn, 2002). The emancipatory work that is critical in schools is addressed with CRT and it works ongoing for a change in the current educational structure. CRT gives scholars the space to do the analysis of centralizing race and deconstructing how law upholds white supremacy in schools, with the aim of moving toward equitable and just learning environments for students. Crenshaw, Gotanda, Peller, and Thomas (1995) outline two central interests that CRT addresses:

> Critical Race scholarship differs in object, argument, accent, and emphasis[;] it is nevertheless unified by two common interests. The first [is] to understand how a regime

> of white supremacy and its subordination of people of color have been created and maintained in America, and, in particular, to examine the relationship between that social structure and professed ideals such as "the rule of law" and "equal protection". The second is a desire not merely to understand the vexed bond between law and racial power but to change it. (p. xiii)

Tenets of Critical Race Theory

There are five main tenets that guide CRT: (a) the permanence of racism, (b) a critique of liberalism, (c) whiteness as property, (d) the interest convergence dilemma, and (e) counter-storytelling and narratives.

Permanence of racism. CRT begins with the idea that racism is "normal not aberrant," a social construction, woven into the fabric of American society. In his groundbreaking book, *Faces at the Bottom of the Well*, Derrick Bell (1992) argues that the legacy of racism in the United States has had an enduring position and that Black people will never gain full access because of it. He goes on to state that:

> Even those Herculean efforts we hail as successful will produce no more than temporary "peaks of progress," short-lived victories that slide into irrelevance as racial patterns adapt in ways that maintain white dominance. This is a hard-to-accept fact that all history verifies. We must acknowledge it, not as a sign of submission, but as an act of ultimate defiance. (p. 12)

CRT analyzes how race is deployed in our daily activities. In other words, it looks at how racist notions are strategically put into action and carried out in everyday life. CRT also analyzes *othering*, which is a way of reifying the positive identity of an individual while stigmatizing an

"other." As DeCuir and Dixson (2004) also point out,

> Permanence of racism suggests that racist hierarchical structures govern all political, economic, and social domains. Such structures allocate the privileging of Whites and the subsequent *othering* of people of color in all arenas, including education." (p. 27)

Critique of liberalism. CRT scholars use race as a unit of analysis in understanding the slow pace in changing legal proceedings for people of color in resolving racial inequalities and gaining citizenship. Liberal legal practice, however, supports these slow changes. CRT scholars have posed critiques of three main liberal ideologies: the notion of colorblindness, the neutrality of the law, and incremental change (DeCuir & Dixson, 2004). CRT scholars agree that racism

needs "sweeping changes, but liberalism has no mechanism for such change" (Ladson-Billings, 1998, p. 12).

Whiteness as property. Taylor (1998) reminds us "democracy in the U.S. context was built on capitalism" (p. 47). For this reason, America's capitalistic system has been a challenge to many minorities because of their lack of power and property. Property rights in the United States are entrenched in racial domination. Cheryl Harris (1995) points out that historically, race was not the only means of subjugating Blacks and Indians: "Rather, it was the interaction between conceptions of race and property which played a critical role in establishing and maintaining racial and economic subordination" (p. 277). In a broader historical context, when conceptualizing property, it "embraces everything to which a man may attach a value and have a

right," which places emphasis on legal rights. Historically, property was not thought of just as the right to material objects and an individual's relationship to those objects. It was also conceived of as "human rights, liberties, powers, and immunities that are important for human well-being including freedom of expression, freedom of conscience, freedom from bodily harm, and free and equal opportunities to use personal faculties" (p. 280). The property functions of whiteness come into being when the law that is in place benefits those holding whiteness with the same privilege granted as those holding property. African Americans in the United States are presented with an interesting predicament as Ladson-Billings (1998) describes: "Not only were they not accorded individual civil rights because they were not White and owned no property, but they were constructed as property!" (p. 15). The economic functions of whiteness as property still exist today. Because whiteness is regarded as the ultimate form of property, many Whites are afforded economic and educational privileges because they possess it. Examples of whiteness

as property are disproportionate home ownership, access to bank loans, job opportunities, and quality education.

Interest convergence dilemma. Critical race scholars argue that the primary beneficiaries to civil rights legislation and most legislation for that matter have been Whites. The interest convergence dilemma holds that the interests of people of color seeking equal opportunity "will

only be granted when the opportunity being sought converges with the economic self-interest of whites" (Bell, 1980, 1992; Tate, 1993). Tate (1993) adds, "No authorization will be given to remedies sought to promote racial equality for African Americans where the remedy sought threatens the economic or social status of the affluent" (p. 17). To further explicate interest convergence, consider the historical court decision, *Brown v. Board of Education* as an example. Some scholars argue that schools were forced to desegregate not because American society wanted to move closer to racial equality (Balkin, 2002; Bell, 1980; Reid, 2006). Instead, American racial politics had become a source of international liability and embarrassment. If the United States wanted to combat communism, they had to look favorably to the international community and remedy explicit racial inequalities at home.

Counter-storytelling and narratives. The integration of experiential knowledge of people of color is also at the heart of CRT scholarship to "analyze the myths, presuppositions, and received wisdoms that make up the common culture about race that invariably render Blacks and other minorities one-down" (Delgado, 1995, p. xiv). Counter-storytelling gives *voice* to people of color. The meta-narratives of equality, meritocracy, objectivity, and capitalism are still deeply rooted in society and very much a part of schooling; they often go unexamined and unquestioned. CRT utilizes the voice of those that are marginalized to break their silence and challenge dominant discourse. Counter-storytelling gives people of color a chance to describe

their own unique oppression within hegemonic structures. When oppressed people are validated through their experiences and stories that counter the dominant narrative, empowerment can be gained. Delgado (1990) also points out "all people of color speak from a base of experience that in our society is deeply structured by racism" (p. 98). Through these experiences, commonalities can be found. Ladson-Billings (1998) also affirms the principal reason that counter-stories are a necessity among CRT scholars: "They add necessary contextual contours to the seeming 'objectivity' of positivist perspectives" (p. 11). Parker and Lynn (2002) further assert that CRT plays an important role because "storytelling constitutes an integral part of historical and current legal evidence gathering and findings in racial discrimination litigation" (p. 10).

Critical Race Theory and Educational Research

Critical race theory has been used by scholars to explicate how race and racism is embedded in our systems of legal practice. Outside of the legal studies, researchers have found great utility of CRT in the field of education. Two notable scholars, Gloria Ladson-Billings and William Tate, are credited with first exploring how CRT could be used in educational scholarship to aid in advancing the field (Ladson-Billings & Tate, 1995). In one of her earlier writings of CRT and education, Ladson-Billings (1998) asserts, "If we are serious about solving these problems in schools and classrooms, we have to be serious about intense study of and careful rethinking of race and education" (p. 22). Ladson-Billings (2000a) also suggests that "although CRT has been used as an analytic tool for understanding the law (particularly civil rights law), as previously noted, it has not been deployed successfully in the practical world of

courts and legal cases of schools" (p. 265). Tate (1997b) argues that because race is deeply embedded in American society, it remains "untheorized as a topic of scholarly inquiry in education" (p. 196). Tate also writes "the significance of race in the United States, and more

specifically 'raced' education, could not be explained with theories of gender or class" (p. 196). Race, gender, and class are usually treated as variables that can be controlled or fixed in research. CRT scholars put race front and center in the analysis of societal structures that impede students from achieving rather than looking at race as a variable factor that can be held constant. Parker and Lynn (2002) explain, "CRT can be used as a tool through which to define, expose, and address educational problems" (p. 7).

Since the initial call by Ladson-Billings and Tate (1995) to foreground race in educational scholarship, CRT has been used in variety of ways. It has been used in educational scholarship to combat dominant narratives of colorblindness by bringing forth voice in understanding the racialized experiences of marginalized racial groups. Such groups are African Americans (DeCuir & Dixson, 2004), Chicanas/Chicanos (Pizzaro, 1998; Solórzano, 1998), and Mexicans (Gonzalez, 1998), to name a few. It has also been used in educational scholarship to advance a critical race pedagogy (Lynn, 1999), investigate non-White children's experience with racism in school and their community (Masko, 2005a, 2005b), explore how CRT may be used as a methodological tool in qualitative research (Bernal, 2002; Chapman, 2007; Duncan, 2002;

Fernandez, 2002; Smith-Maddox & Solórzano, 2002; Solórzano & Yosso, 2002), and examine how African American students experience and respond to racial microaggressions on a college campus (Solórzano, Ceja, Yosso, 2000).

In qualitative educational scholarship, CRT has also been deployed as an analytic tool in revealing the complicities of race in affirmative action university admission policies (Parker, 2008), analyzing the experiences of African American students at a White private school (DeCuir & Dixson, 2004), and uncovering how it is operationalized in institutions such as an urban middle school (Masko, 2008). Parker (1998) used CRT to analyze the affirmative action

policies of an educational legal case. He conducted a study in which he analyzed the underlying assumptions of affirmative action in the legal case *Hopwood v. Texas* through a critical race lens. The court's ruling determined that race should not be used as a determining factor to attain diversity. Using CRT in the analysis, Parker determined that affirmative action was seen as allowing less-qualified applicants admission to a university law school over White Americans rather than viewing it as mechanism for increasing the diversity in admissions.

Masko (2008) also conducted a 2-year ethnographic study in an urban middle school in a midwestern community. Masko (2008) explored the ways in which students and teachers related to each other and constructed meanings both in the classroom and in the school, related to race. Masko found that resistance played a major role in the school at the administrative level,

curriculum level, and among students. At the administrative level, administrators protested practices they believed would not be beneficial to the student population. This was seen in the administration's response to job cuts at the school of important staff and also their response to what was deemed as indifference by the district's central office to initiatives made to close the achievement gap among students at the school.

At the curriculum level, resistance was seen in the culturally responsive nature of the curriculum of one teacher. Her resistance was actualized in the types of literature she selected for one of her classes. The teacher also required students to bring articles they felt were more representative of their lives. Furthermore, for an honors curriculum class, which attracted predominantly White students, the teacher made a substantial effort to seek out more students of color who also met requirements for the class. As a result of the teacher's effort, there was a more racially diverse student body in the honors class. Masko also found resistance within students. Students resisted the society's expectations for students of color, who were perceived

as low achievers. Some students did this by consciously making an effort to stay in school and persevering rather than selling drugs or dropping out like some of their peers. Some students also resisted multicultural education. One White student resisted the emphasis placed on race and racism in education and at her school, which regularly celebrated events like Black History Month. She believed that "we should be treated the same way," by also emphasizing other

groups that have discriminated against. In this sense, the student failed to see the significance of race and how it played out in her ideology.

In mathematics education, some researchers have used CRT to convey counter-narratives of students who succeeded despite dominant narratives of underachievement that prevail in the field. Researchers have used CRT to examine African American students' progress in mathematics over the last 50 years (Snipes & Water, 2005) and to tell the stories of successful African American males in mathematics (Berry, 2005, 2008; Stinson, 2004).

Snipes and Water (2005) conducted case study research in which they interviewed a former state mathematics consultant who described his experience in mathematics education in North Carolina during the 1950s to 1980s. During the participant's tenure as state mathematics consultant, he observed several occurrences of institutional and structural racism. Examples of these included inadequate school resources provided to Black students, tracking students, and low expectations of teachers teaching Black students. The participant also pointed out that before integration, high schools with predominantly Black students required students to take a minimum of Algebra 1 before graduation. After integration, the attitude across high schools in the state was that Black students were not smart enough to take mathematics courses that would prepare them for college. Finally, the study also revealed that after integration, the number of teachers who

prioritized nurturing Black students sharply declined. Although Black students were now in schools with better resources, their intelligence was constantly doubted.

Drawing from a larger study he conducted, Berry (2005) tells the stories of two African American male middle school students who experienced success in mathematics. Utilizing CRT for its role in illuminating how race and racism plays a part in shaping schools and schooling practices along with descriptive portraits, Berry investigated the barriers these African American male middle school students encountered and how they overcame these challenges to do well in mathematics. He found that the parents of the students provided early academic experiences to excite and expose their sons to learning in early elementary grades. These early experiences included a number of educational games and activities in the students' early years. Berry also found that the students had experienced substantial discrimination. The students in the study were misdiagnosed with learning disabilities and were wrongly placed in classes that did not challenge them academically. As a way of fighting against these challenges, the parents played the role of advocates for their sons. Berry also found that in addition to strong support systems, the students were self-empowered and highly motivated to be successful in school and mathematics.

Berry (2008) then looked at all eight successful African American males students from

his larger study, while telling the story of two of them, along with the barriers they faced in gaining access to high-level mathematics courses. Success was defined as enrollment in Algebra 1 in middle school. Berry's study revealed that the boys embodied alternative identities that enabled their persistence in school and coping with peer pressure. Identities the boys embodied were co-curricular and special academic program identity, religious identity, and athletic identity. In this sense, the boys' participation in extra-curricular activities reified their idea of

smartness. Further, seven of the eight boys demonstrated strong spiritual faith and participated regularly in church activities. Finally, five of the eight boys participated in sports throughout the year. Participating in athletics kept the boys focused while they were not in school.

Stinson (2004) conducted a qualitative action research study on the schooling experiences of African American male students who fully participated in school, academics, mathematics, and who were identified as having agency. Agency was defined as the ability of participants to accommodate, resist, or reconfigure the available sociocultural discourses that surrounded African American males in order for them to effectively negotiate these discourses in their pursuit of success. Stinson used a critical postmodern theoretical framework, which drew upon a number of theories, including CRT. He utilized CRT to understand how the discourse of race and racism operated within U.S. social structures. Through the use of CRT, Stinson found that participants identified stereotypical discourses, which suggested that African American females

were smarter than Black males. Participants believed that the pervasiveness of this stereotype resulted in lower school and academic expectations of Black male students. Participants suggested that teachers and the African American community embraced lower expectations of Black male students. Participants also challenged widely held negative images of African American males who rejected and were deficient in school.

Common themes among these studies were one of doubt in the abilities of Black students to not only achieve in mathematics, but also to persist in school. The studies in mathematics education were highlighted because they underscore the importance in illuminating the stories of students who, contrary to a dominant discourse of failure, can succeed in mathematics and do well.

Educators have questioned traditional methods of teaching and learning mathematics and the ways in which knowledge is transferred to students through those approaches. They have even suggested that types of learning situations define the types of knowledge that students acquire (Boaler, 1999; Boaler & Greeno, 2000). For mathematics specifically, it is the "practices of learning mathematics [that] define the knowledge that is produced" (Boaler & Greeno, 2000, p. 172). In this sense, situations and activities in practice are integral to cognition and learning

(Brown, Collins, & Duguid, 1989). Social theories of learning espouse that learning occurs by doing or through practices in social activity (Lave & Wenger, 1991; Wenger, 1998).

In a broader study of identity, scholars have considered various ways in which individuals come to view themselves. Identity-related topics have been explored primarily in the psychological domain (Phinney, 1990; Steele, 1997; Steele & Aronson, 1995). From a psychological perspective, there is also an extensive body of literature that explores how racial identity is developed (Cross, 1991; Fordham, 1996; Hale-Benson, 1994; Helms, 1990; Sellers, Smith, Shelton, Rowley, & Chavous, 1998; Tatum, 1997; Welch & Hodges, 1997). It is, however, beyond the scope of this discussion to consider identity from a psychological perspective. Departing from the view that identity and learning can only be explained by cognitive theories, I support scholars who have taken a more situated perspective (Cobb & Hodge, 2002; Lerman, 2000). Identity in this sense is defined as the negotiation between learners and the social contexts in which they participate in that influence how they view themselves. Some scholars have described the types of identities that emerge from these social contexts. These types of identities include social identities (Askew, 2007; Carr, 2001; Fernie, Davies, Kantor, & McMurray, 1993), learner identities (Weil, 1986), and even a mathematics identity

(Martin, 2000, 2007). Wenger (1998) provides a useful framework for understanding the connection between learning and identity formation that takes place within a social context

through communities of
practice.

Communities of Practice, Learning, and Identity Formation

Communities of practice defined. A community of practice is a collective group, unified by common interests where members interact regularly in order to create and improve what they learn and share over time. Although various areas of the social sciences use and apply this concept, its origins and utility are found in learning theory. Theories of learning enable an understanding of how learning takes place in humans from a behaviorist, cognitivist, or constructivist perspective, to name a few. Etienne Wenger, a Swiss educational theorist, in collaboration with anthropologist Jean Lave, is noted for first developing the term *communities of practice* (CoPs) (Lave & Wenger, 1991; Wenger, 1998). Some scholars have claimed that both Lave and Wenger (1991) and Wenger (1998) cast a similar, yet "characteristically different [concept] of 'community of practice'" (Kanes & Lerman, 2008, p. 304). For the purposes of this

discussion and to provide clarity, I will primarily focus on Wenger's (1998) view of CoPs.

The theory of CoPs espouses that learning requires extensive participation in a community whose members are engaged in a set of relationships over time. Wenger (1998) explains that CoPs exist all around us; they are an important part of our everyday life, and individuals are part of a number of them both implicitly and explicitly. CoPs range from formal to informal; some individuals are core members, and others hold peripheral membership.

Clarifying the definition of CoPs, Wenger, McDermott, and Snyder (2002) explain:

> As [members] spend time together, they typically share information, insight, and advice. They help each other solve problems. They discuss their situations, their aspirations, and their needs. They ponder common issues, explore ideas, and act as sounding boards. They may create tools, standards, generic designs, manuals, and other documents—or they may

> simply develop a tacit understanding that they share. However they accumulate knowledge, they become informally bound by the value that they find in learning together....Over time, they develop a unique perspective on their topic as well as a body of common knowledge, practices, and approaches. They also develop personal relationships and established ways of interacting. They may even develop a common sense of identity. (pp. 4-5)

In order to distinguish CoPs from other types of sustained social relationships, Wenger (1998) lists fourteen indicators that a CoP has been formed. They are the following:

> 1. Sustained mutual relationships – harmonious or conflictual 2. Shared ways of engaging in doing things together 3. The rapid flow of information and propagation of innovation 4. Absence of introductory preamble, as if conversations and interactions were merely the continuation of an ongoing process 5. Very quick setup of a problem to be discussed 6. Substantial overlap in participants' descriptions of who belongs 7. Knowing what others know, what they can do, and how they can contribute to an enterprise 8. Mutually defining identities 9. The ability to assess the appropriateness of actions and products 10. Specific tools, representation and other artifacts 11. Local lore, shared stories, inside jokes, knowing laughter 12. Jargon and shortcuts to communication as well as the ease of producing new ones 13. Certain styles recognized as displaying membership 14. A shared discourse reflecting a certain perspective on the world. (pp. 125-126)

For the purpose of this discussion, I limit my focus to three fundamental but interrelated structural elements that can be used to differentiate CoPs from other closely related social

groups. *Structural elements of communities of practice*. There are important characteristics that all CoPs embody. CoPs contain a combination of three fundamental elements: a domain of interest or *mutual engagement*, a community or *joint enterprise*, and a focus on practice or *shared repertoire* (Wenger, 1998; Wenger, McDermott, & Snyder, 2002). Without these three critical interrelated elements, CoPs could not exist.

A domain of interest indicates what the group is about. The domain gives the group a legitimate identity, which reifies its purpose and value. This is done by sharing a mission statement or by having common purpose in the group. Domain is connected to mutual engagement. Mutual engagement describes the functionality of a CoP. Through mutual engagement the community is defined. It binds members together into a social entity. The practice of the CoP is not just an abstract or static concept, nor is it just in theory or in one's mind. Mutual engagement represents a practice in which individuals come together to engage in some negotiated set of actions. Furthermore, through mutual engagement in a community of practice each participant "finds a unique place and gains a unique identity, which is both further

integrated and further defined in the course of engagement in practice" (Wenger, 1998, p. 76).

> The community tells how the group functions. When the community is strong, it fosters

interactions and relationships based on a mutual respect. Community is linked to joint enterprise. A joint enterprise is constantly renegotiated; it maintains the CoP and keeps it together. Through joint enterprise CoPs define their collective vision or purpose. As Wenger explains, there are three key features of a community that keep it intact. First, it is the collective process of mutual and complex engagement. Second, it is defined by the pursuit of its participants and belongs to each of them. Finally, its participants, whereby all members are accountable and are an essential part of the CoP, mutually constitute the joint enterprise.

The practice of the group indicates the types of things the group produces. Through practice, the CoPs use specific ideas, tools, and ultimately produce knowledge. Practice is connected to shared repertoire. Shared repertoire represents how CoPs evolve over time and the capabilities they produce. Through the joint pursuit of an enterprise, resources are used in specific ways that become important in negotiating new meaning. The shared repertoire of CoPs

includes the common language, routines, and ways of doing things that its participants engage in. Moreover, through a shared repertoire, the participants are able to articulate their forms of membership and their identities as members.

There are also other important structural elements that CoPs possess. CoPs are diverse and take on a variety of forms. CoPs can range from small to big, long-lived to short-lived, collocated to distributed, homogeneous to heterogeneous, inside to across boundaries, spontaneous to intentional, and unrecognized to institutionalized (Wenger, McDermott, & Snyder, 2002). These distinctions are useful so that CoPs can be easily recognized.

> CoPs range in size. Some can be quite large, and others are small. Some CoPs have few

members, with very specialized expertise, and others have large groups located in various geographic regions where all members are encouraged to participate. CoPs can also exist over long periods of time. Mathematics educators, for example, have a history of practice that span several decades. Then there are CoPs that have only existed for just a few years. Because CoPs require regular interaction, some of them operate among individuals who live within close proximity, whereas others are across many cities. Communication in these CoPs can be transmitted through emails, websites, phone, and face-to-face contact. The key here is that, no matter where the CoPs are located, there is a shared practice. No matter where the CoPs exist, there is a common understanding, a common set of issues that the community deals with, and a common way of looking at problems. CoPs can also be composed of individuals with similar

backgrounds, whereas others have individuals that are more diverse. If the backgrounds of the individuals in the CoP are not similar, then members are unified through a common issue or concern. This unification helps to build the community's shared practice. CoPs can emerge

naturally and informally from repeated conversations in the group. They can also be intentionally

created for a specific purpose. Some CoPs come together spontaneously because of a specific

need that must be addressed. Other CoPs are very structured in their formation and conduct themselves with rather specific protocols, like agendas, meeting schedules, or meeting minutes. Finally, some CoPs can go unrecognized, whereas others can be institutionalized. Because some CoPs happen naturally, there may not be public acknowledgement of their existence. Despite this lack of awareness, some CoPs that go unrecognized develop shared knowledge that can inform the practice of a community at a later time. CoPs that are formally established are used in some organizations because their presence serves a specific need of a broader community.

Modes of belonging in identity formation. An integral component of any community of practice and an important part of learning is construction of identities. Participants in a community build their image of how they see themselves through the positions they hold, which greatly influences their construction of identity. In regards to identity formation and learning, Wenger offers three distinct modes of belonging: engagement, imagination, and alignment.

Similar to mutual engagement, engagement describes interactions as an ongoing collaboration with members in communities that can change. Engagement involves three processes: ongoing negotiation of meaning, the formation of trajectories, and the unfolding of

histories of practice (Wenger, 1998, p. 174). The simultaneous interaction of these three processes allows engagement to become a form of belonging, which is how identity unfolds. More important, through engagement in a community a shared reality is created and aids in constructing identity. Engagement embodies natural boundaries because of limitations of time to participate in the community, activities one can become involved in, and relationships that members of the community develop with one another. Because of these limitations, engagement often has both strengths and weaknesses.

In a community, imagination is the ability of members to create new images of themselves beyond time and space. Imagination is a creative mental process that can be thought of as the ability to dream about what is possible for oneself and what one is capable of becoming. Unlike engagement, which is concretized in creating a shared reality, imagination supersedes direct interaction and is another form of constructing a shared reality. Our imaginations allow us to locate ourselves in the social world, past, present, and future. In this mode of belonging, the scope of our reality is expanded along with our identity.

Alignment is the mode of belonging in which all efforts, such as energies, actions, and practices, come together to produce coordinated activities. Alignment allows participants in a community to play their unique roles in finding common ground. Participants are able to arrange their practices within the guidelines and expectations of the community to maintain alignment.

Finally, alignment necessitates very precise forms of participation and structure.

Each of the three modes of belonging has its share of shortcomings. One mode should not be thought of as better than another. Each should be thought of as working cohesively with the others in the construction of identity. To better understand the processes of identity formation, consider Figure 2.1. It shows the interconnection of the three modes of belonging, and the possible elements of each that are negotiated.

Identification and negotiability in identity formation. In addition to the modes of belonging, Wenger (1998) also clarifies that identity formation is an interconnected process of identification and negotiability. Through this duality, "our identities form in this kind of tension between our investment in various forms of belonging and our ability to negotiate the meanings that matter in those contexts" (p. 188). In other words, there is mediation between one's ability to identify or belong and one's understanding of the degrees of what it means to belong in various

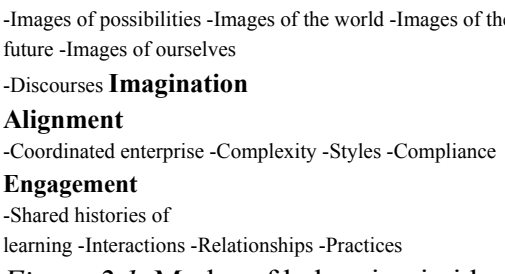

Figure 2.1. Modes of belonging in identity formation. (from Wenger, 1998, p. 174).

contexts. Wenger (1998) defines identification as the "process through which modes of belonging
become constitutive of our identities by creating bonds or distinctions in which we become

invested" (p. 188). In other words, identification can be thought of as the process of becoming a member of the CoP, possessing a personal and deep level of allegiance to the CoP, and engaging in activities that signal membership in that community. Identification itself is dynamic and complex, a generative process that changes. Identification should not be thought of as just how individuals relate to one another. It is also the interplay of all aspects of what makes CoPs possible; from the participants to the enterprises that participants are a part of.
Negotiability is defined as "the ability, facility, and legitimacy to contribute to, take responsibility for, and shape the meanings that matter within a social configuration" (Wenger, 1998, p. 197). In other words, negotiability is a process of understanding what sense is made of the meaning in the things we place upon ourselves and that are placed on us. Wenger also suggests that negotiability is "shaped by relations of ownership of meaning" (p. 200), which

enables meanings to embody various degrees of value. Identification and negotiability interact with the modes of belonging to aid in the understanding of identity construction.

Trajectories and learning in CoPs. In CoPs, identity is produced from the very act of participation and the meaning that is created for oneself within that participation. This work is far from static – it is ongoing. One might even say that it is a "constant becoming" (Wenger, 1998, p. 154) and in some respects something that we reexamine throughout the course of our lives. Wenger suggests that as we go through various forms of participation, our identities formulate trajectories inside and throughout the CoPs. Wenger's (1998) notion of a trajectory should not be characterized as a charted or predetermined path. It should however, be considered as "a continuous motion – one that has momentum of its own" (p. 154). In CoPs, as members move from peripheral participation to full membership, they might find themselves on various trajectories such as peripheral trajectories, inbound trajectories, insider trajectories, boundary trajectories, or outbound trajectories.

Peripheral trajectories seldom lead to full participation by members in the CoPs. This decision may be voluntary or required, but it still gives individuals access to the community whereby some aspect of their practice can influence their identity. An inbound trajectory suggest that individuals seeking membership in CoPs become part of the community with the intention of becoming full participants over time. The identities that individuals formulate are interconnected with the participation they hope to have in the future, even though their position as newcomers may only afford them marginal participation at the time. An insider trajectory allows individuals to go beyond full membership. Their practice allows new innovation and constant reexamining of their identities for something new and different. Boundary trajectories enable connections to be made between other CoPs along with the identities that must be sustained in these spaces.

Finally, an outbound trajectory in some sense prepares individuals for their role in the community as they decide to transition from the community. Identities are then contingent on how they view themselves and the world as they make these transitions.

The relationship of identity and learning are important here because as we reexamine the identities we create for ourselves, it defines how we engage in our practice. In this sense, engagement is not fixed, but can shift as our identity shifts. As a consequence, what is prioritized in our learning is contingent upon what trajectory we find ourselves on. Wenger (1998) points out that “a sense of trajectory gives us ways of sorting out what matters and what does not, what

contributes to our identity and what remains marginal" (p. 155).

Wenger's framework provides a way of considering the practice of doing mathematics in an out-of-school context. Wenger's explanation of CoP, modes of belonging, and identification and negotiability, and trajectories were important in framing this study because they provided a lens for understanding what to look for in determining what identities were constructed in workshop training with CMLWs and MLWs at a particular training hub. It also enabled me to understand what kinds of negotiation took place in the Flagway number categorizations. In the next section, I discuss how CoPs have been used in research.

Research on Communities of Practice and Mathematics in a Social Context

Communities of practice are an emergent topic in the research literature; however there are very different accounts by scholars of what CoPs actually mean. Not all scholars refer to Wenger's (1998) work or even to Lave and Wenger (1991) in their conceptualization of CoPs. The majority of the studies that I highlight here draw from Wenger's (1998) notion of CoPs. Because the body of mathematics education research on CoPs is growing, I do cite some studies that draw from Lave and Wenger (1991) since the two perspectives are related.

CoPs in research. From their inception, CoPs have been used primarily to study how commercial organizational systems function. In an effort to further understand and extend the CoP framework in this area, researchers have worked to broaden its definition (Manville &

Foote, 1996; Orr, 1990; Seely Brown & Duguid, 1996; Stewart, 1996) and to investigate how it might be applied across international boundaries (Hildreth, Kimble, & Wright, 2000). Beyond commercial systems, researchers have made efforts to expand their understanding of CoPs in other domains such as education and the other social sciences.

In the social sciences, CoPs have been used to further studies in sociolinguistics, with special attention given to research on language and gender (Bergvall, 1999; Bucholtz, 1999; Eckert & McConnell-Ginet, 1999; Ehrlich, 1999; Freed, 1999; Holmes & Meyerhoff, 1999; Meyerhoff, 1999). Research on CoPs in educational organizations has been conducted to better understand the practices that influence daily life in workgroups of deaf academics in a university setting (Trowler & Turner, 2002). Educational anthropologists who study teaching and learning both inside and outside schools have looked at understanding the knowledge that is brought to a CoP of novice child-care teachers (Bradley, 2004). In medical education, CoPs have been used as a framework for analysis in clinical teaching and learning for students training to be health professionals (Egan & Jaye, 2009). In the virtual domain, scholars have used the framework to ask whether online CoPs constitute a community of learners (Reimann, 2008). In teacher education, CoPs have been used extensively to understand how teachers come to learn and practice in social contexts. CoPs have been used to understand how to prepare and support literacy educators in a single diverse community (Au, 2002), to advance an educational reform agenda (Lieberman & Pointer Mace, 2008), and to document the changing roles of teachers utilizing technology in classrooms (Hartnell-Young, 2006). CoPs have been used as a strategy

for effective professional development in the training of teachers to connect and interact with each other (Looi, Lim, & Chen, 2008). Professional developer training teachers have used CoPs to explore a nontraditional professional development process, which involved a small group of teachers and a professional developer working as a collaborative community (Fougler, 2005).

CoPs in Mathematics Education Research

> There has been a growing effort to extend Wenger's work on CoPs in mathematics

education. In mathematics education, CoPs have been used to describe and clarify teacher learning (Adler, 2001; Graven, 2002, 2004; Stein & Brown, 1997) and student learning (Boaler, 2000; Santos & Matos, 1998). Further, CoPs have been used to look at how mathematics learning communities are developed in primary classrooms (Price, 2003) and to study identity and how it plays its role in learning in the mathematics classroom (Cobb & Hodge, 2002).

Research on teacher learning. Adler (2001) interrogated the teaching dilemmas that 6 teachers in multilingual secondary level mathematics classrooms negotiate within a post-apartheid South Africa. Because of multilingual backgrounds that both teachers and learners bring to mathematics classroom in South Africa, Adler was particularly concerned with how the mathematics teachers mediated their practices in these communities. Although the study was grounded in ethnographic methodology, Adler used social practice (Lave & Wenger, 1991) and

socio-cultural theoretical perspectives to understand emerging issues around teaching and learning that arose in these multilingual classrooms. In Adler's findings, she describes these dilemmas as code-switching, mediation, and transparency and argues that teaching dilemmas in general are situated and have contextual meaning. Adler found that teachers expressed a variety of issues related to the three dilemmas. For instance, because the official language is English in South Africa and for learners whose main language was not English, many teachers expressed

occasional difficulty explaining mathematics. Further, learners had difficulty articulating what they understood in mathematics in English. Adler also found that because of the changing discourse of codeswitching in these communities, mathematical meaning of very explicit mathematics terms like *not more than* and *at least* were negotiated between everyday and mathematical language, and between verbal and symbolic forms. Adler also found that teachers had to negotiate the explicit mathematics language of the classroom. In this sense, language became the focal point in the mathematics classroom, as opposed to the mathematics that was trying to be taught. Through this negotiation, the mathematics often became lost.

Graven (2002) conducted a longitudinal study in South Africa in which she looked at what mathematics teachers learn through participation in a community of practice stimulated by a new curriculum change effort. One important finding of the study was that through the CoP, teachers identified new roles for themselves and saw their learning as another way of connecting

to their practice. For example, before membership in the mathematics community, one teacher identified herself as a music teacher and after membership, identified herself as a "maths teacher." Graven (2002) also found that through membership in the mathematics community teachers' identity grew stronger.

Drawing from the same longitudinal study, Graven (2004) looked at the primary role of confidence in understanding the learning of teachers that occurred within a mathematics in-service program prompted by curriculum change. Although not a focus in Wenger's work, it was Graven's goal to bring confidence to the fore to extend Wenger's framework. Graven argues that confidence too is a necessary component of learning, just like meaning, practice, identity, and community. In the study, she found that confidence was both a product and process in the mastery of becoming and being a professional mathematics teacher. That is, confidence was

necessary for the teachers to move from being teachers of mathematics to becoming competent and confident mathematics teachers. Graven also recommends extending the literature on confidence to a social perspective, because confidence is primarily seen within a psychological domain. *Research on student learning.* Drawing from Lave and Wenger's (1991) work, Price (2003) studied how a learning community in the classroom could be established between a

teacher and 27 primary school children learning addition. The study took place in England with students 4 to 9 years old. Price found that both teacher and the students created an environment in the classroom where making mistakes and just trying to do the mathematics was accepted. Price found no incidents where the teacher explicitly told students that they were doing the mathematics wrong. When wrong answers did surface, the teacher asked the class whether they agreed with the student giving the answer. She then encouraged the student to rethink their response by telling them that their answer was "nearly" correct. Students were also encouraged to use mathematical discourse during lessons. Price also found that because of the establishment of the community, the result was a positive effect on the students' motivation to do mathematics and how they viewed mathematics overall. Further, Price also found that the students wanted to gain mastery in mathematics and to become more able learners.

Cobb and Hodge (2002) presented a discussion on a relational perspective of cultural diversity and equity using CoPs as a possible analytic tool to foreground local communities outside of school, such as home communities. Further, as another possible analytic tool, notions of discourse (Gee, 1997), which is the daily language produced by various types of communities, were also suggested since it can distinguish between broader communities within society. Cobb and Hodge drew from these two perspectives and suggested that taken together, they might be a

fruitful way of analyzing the "classroom microculture [if] the teacher and students in a classroom

necessarily constitute a community" (p. 273). Cobb and Hodge also suggest that CoPs and discourse might be useful in analyzing mathematical practices in the classroom because of the cultural capital that is negotiated between members.

Identity and mathematics in socio-cultural practice. There is a common belief that the primary context where students acquire the mathematics they learn is in schools. Challenging this notion, researchers have studied learning as participation in daily cultural practices and out-of-school contexts to better understand how adults and students acquire basic mathematical concepts. In earlier studies, researchers examined mathematics in everyday situations, like the use of arithmetic (de la Rocha, 1985; Murtaugh, 1985; Scribner, 1984), geometry (Millroy, 1992), rational number concepts (Carraher, 1986), and measurement (Masingila, 1994). Researchers have considered mathematics learning in out-of-school activities like candy selling (Saxe, 1988a; 1988b), carpet laying (Masingila, 1994), playing educational games (Saxe & Guberman, 1998), and playing basketball and dominoes (Nasir, 2002). Common themes of research in this area suggest the following as it relates to learning in out-of-school situations: (a) the mathematical knowledge that individuals bring to out-of-school activities are more than adequate; (b) mathematical procedures are created on the spot as needed; (c) mathematical activities are structured in relation to the ongoing activity one engages in; and (d) identities are always in constitution as individuals engage in practice.

Carraher (1986) conducted a comparative study between students and construction foremen in Brazil to examine their knowledge of scales, which is based on proportional

reasoning. The students selected for this study were seventh graders attending a middle school that served middle-class to affluent families. The construction foremen selected for the study

acquired informal knowledge about their profession either by some apprenticeship training or through friends. Four types of scales were presented: two that were commonly used in construction, and two that were not. Carraher found that the students had very strong syntactic abilities in solving scale problems, but had a rather difficult time making meaning of the work they carried out. Furthermore, a majority of the students in the study created their own methods for solving problems. The construction foremen, on the other hand, had no problems in understanding the meaning of the problems and were able to work more efficiently even when utilizing the same problem-solving strategies as the students used. They used hypothesis testing, and those who used it were very familiar with the scales with which they worked. Because of these differences in meaning, Carraher suggests that students learning more about proportions should engage in deeper reflection so that they will have a more rounded understanding of the concept. Saxe (1988a, 1988b) conducted a study to investigate how daily activities performed by children enabled their numerical understanding. In his work with child street vendors in Brazil, Saxe investigated the mathematical goals they developed in their day-to-day practice of handling

currency. As a method for understanding these goals, Saxe (1988a) investigated the sellers' practice- linked understandings of representing large numerical values, arithmetical manipulation of numerical values, comparison of ratios, and adjustment for inflation in wholesale to retail markups. Saxe (1988b) also looked for selling strategies and solutions among sellers and two groups of nonsellers (urban and rural). In support of the themes mentioned earlier, findings from this study suggested that sellers acquired mathematical knowledge needed to engage in practice-linked tasks. Saxe also found that despite the limited availability of schooling in Brazil, sellers and both groups of nonsellers were able to organize their currency system to represent large

numerical values in adequate ways. This study showed the limited connection between the school-based mathematics and the out-of-school mathematical activities of sellers. The study also supported the rationale for a stronger connection between the mathematics that is learned in schools and in out-of-school activities.

In a later study, Saxe and Guberman (1998) examined the mathematical goals that elementary school age children formulated as they engaged in joint play of an educational mathematics game they created called Treasure Hunt. Saxe and Guberman compared dyads of different ability groups in game play to understand the types of goals children set up when solving mathematical problems. They found two principal forms of social interaction that took place during game play by students: direct assistance and thematically organized assistance. In

game play, students directly assisted their opponent in varying degrees to solve problems. Students also achieved thematic goals during game play when solving joint problems. In that way, students relied on prior mathematical knowledge and invented character roles during game play to accomplish certain tasks. Because the game primarily dealt with purchase transactions, children assumed specific roles as shopkeepers and customers to accomplish mathematical tasks. Saxe and Guberman chose three samples to conduct their study: two ability groups, similar and mixed, and one control group. Their primary interest was in the mathematical aspect of the children's work during game play. Identity themes emerged in their findings but were not explicitly addressed through the roles that students invented. The themes were seen in the goals students created to solve problems by *becoming* or *adopting* character roles to complete tasks during game play.

Aligning with earlier themes described and using Wenger's (1998) framework to

explicate modes of belonging in identity formation, Nasir (2002) describes findings from two

studies she conducted on mathematics learning, both focused on African American students engaging in out-of-school cultural practices of dominoes and basketball. Nasir's research also builds on Saxe's work, considering the practice-linked goals, mathematical goals, and identity that students have in their practice. Nasir examined the relationship between culture, practice-

linked goals, and identity formation of students in these two contexts. In both basketball and dominoes, the findings suggested that players required a competency in mathematical operations, probability and logic, and statistics to make calculated moves in each game. More important, the mathematical goals of the students playing both games depended on their age and level of engagement in each game. The identity of students playing dominoes shifted because of the changing nature of their engagement. This shift in identity was related to the new ways each group participated in the practice of dominoes. The students in elementary school and high school, made different investments in how they played dominoes. The high school players had a more rapid playing style and paid more attention to the game than the elementary school players. The high school students moved between game and nongame talk while playing dominoes. Using the playing skills they developed, the players also developed respect, and in some cases disrespect, for their partners and opponents over time. The elementary school player's relationship with one another was connected to how they saw each other outside of the game. Students with "large amount of power and authority outside the game context" (p. 229) were deferred to consistently. Furthermore, because the elementary school students were new to the game of dominoes, there was little shared experience with the game.

For students playing basketball, all modes of belonging, engagement, alignment, and imagination changed, which also reflected changes in players' practices. For engagement, students playing basketball were committed to playing the game. They practiced diligently, were

dedicated to winning, and maintained professionalism. For alignment, players referred to themselves as "ballers" and kept current with professional players by reading magazines, watching games on television, and keeping track of their score statistics and major trades. For imagination, high school players connected themselves to those in their community that were afforded the opportunity to play basketball in college. All of these students, with the exception of one, saw themselves playing basketball in college.

These studies were considered because they focused on out-of-school contexts where mathematics is learned. These contexts were related to my study. Although identity was not a focal point in all of these studies, some studies did reveal that adapting identities could be a consequence of engagement in social practice. Further, the present study built upon Nasir's work, where she investigated the identities adopted as a consequence of students' engagement in an out-of-school context where mathematics learning took place.

Research on Functions and Prime Decomposition

Student Understanding of Functions

Functions can be seen in various places across mathematics from elementary concepts to more advanced topics (Lloyd & Wilson, 1998; Szydlik, 2000; Williams, 2001). There is a consensus among researchers that the concept of function is essential to teach and that students

must learn it in order to deepen their understanding of mathematics (Dubinsky, 1993; Kalchman & Koedinger, 2005; Knuth, 2000; Lacampagne, Blair, & Kaput, 1995; Leinhardt, Zaslavsky, & Stein, 1990). While the research literature on functions is quite vast, my aim was to organize this review according to what the research has informed us on students' understanding of functions and the types of functions that have been studied.

Because of its complexity, many students have difficulty in understanding function. Leinhardt, Zaslavsky, and Stein (1990) cite three reasons why functions are difficult for students to master. They suggest that the concept of function is often connected to other mathematical concepts; it draws from various fields of mathematics; and functions can have multiple representations (Dreyfus & Eisenberg, 1982). Kalchman and Koedinger (2005) also cite reason why they believe students to do not understand functions. They suggest that when "students' conceptual understanding and metacognitive monitoring are weak, their efforts to solve even fairly simple algebra problems can, and often do, fail" (p. 353). Sajka (2003) offers other reasons for students' difficulty in understanding. Functions have an inherent abstractness in their notation, and there are restrictions in symbol representation when teaching and limitations in the mathematical tasks are offered at schools.

Researchers have proposed that students have varied understandings of the definition of

function. Although they may be able to explicitly give a definition of a function, students have a rather difficult time applying that definition to graphs (Vinner, 1983). Misunderstandings are also seen in graphical representations when students are asked to classify functions in various tasks (Vinner, 1983; Vinner & Dreyfus, 1989). This research also suggests that students have limited ideas of what graphs of functions can represent. Often, students fail to recognize graphs of functions that are unusual or that they have not come across before.

Researchers have also reported that students have a difficult time understanding correspondence (Lovell, 1971). Students seldom understand correspondences that are different from one-to-one. Many students only consider examples of functions with the one-to-one property. They are often confused by the concept of many-to-one and one-to-many correspondences. Researchers have suggested that this confusion may stem from a lack of