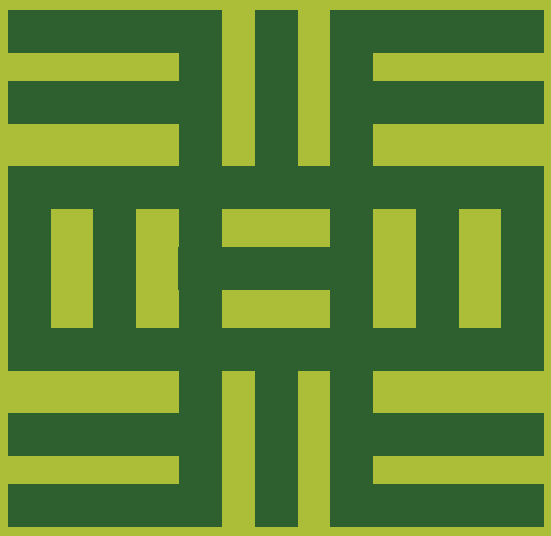
Road Coloring



Teacher's Guide
2017 - 2018

THE ALGEBRA PROJECT

Road Coloring

Table of Contents

Curriculum Goals

Broad Unit Goals

Essential Questions	Enduring Understandings
• What is a function? • How can functions be represented? • What real life occurrences do functions model?	• Functions are mathematical relationships in which every input has a single output. • Functions can be represented in a variety of ways. • Matrices hold data in a specific way that can be operated on. • Inverses of functions return data to the original state before any action was taken.

Key Knowledge and Skills to be Gained

Students will know…	Students will be able to…
• Key Terms: function, arrow diagram, directed graph, ordered pairs, coordinate plane, • The One-to-One Rule • Function Notation • Vertical Line Test	• Draw correct models for functions including: arrow diagrams, directed graphs, matrices • Differentiate between relations that are functions and those that are not and explain why • Plot functions on Cartesian planes • Use graphing calculators to plot functions • Perform matrix arithmetic • Compose functions • Find inverses of functions

Standards Addressed

Section	Standards
	• HSF-IF.A Understand the concept of a function and use function notation.
	• HSF-IF.C Analyze functions using different representations.
	• HSF-IF.B Interpret functions that arise in applications in terms of the context.
	• HSN-Q.A Reason quantitatively and use units to solve problems.
	• HSN-VM.C Perform operations on matrices and use matrices in applications.
	• HSF-BF.A Build a function that models the relationship between two quantities.

Curriculum Overview and Pacing Guide

Curriculum Overview (10 Days)

Day	Curriculum Section Title	Objectives/Assessments
1	Building Cities	• Build city • Analyze and record everything about the cities
		• City diagrams worksheets
2	Naming the Roads	• Color city diagrams correctly.
		• Colored city diagram worksheets
3	Solving the Puzzle	• Solve puzzle
		• Puzzle solving worksheets • Team questions
4	Models and Representations	• Create directed graph and arrow diagrams of cities. • Record movements in the Road Coloring Problem. • Draw arrow diagrams and road subgraphs of cities
		• Directed graph and arrow diagram worksheets • Arrow diagrams and subgraphs worksheet
5	Representations with the Coordinate Plane	• Create lists of, and plot, ordered pairs • Complete multiple representations of cities • Apply information from multiple representations to the Road Coloring Problem
		• Arrow diagrams and ordered pairs worksheet • Multiple representations worksheet • Vertical Line Test Worksheet
6	Functions	• Determine whether a representation of a city represents a function. • Write the domain and range of a function.
		• Function worksheet

7	Matrix Representations	• Create a matrix representation of a city.
		• Matrix representation worksheet
8	Function Composition	• Compose functions from directed graphs, arrow diagrams, and ordered pairs.
		• Function composition practice sheets.
9	Function Notation	• Convert directed graph, arrow diagrams, and ordered pairs into function notation.
		• Function notation practice sheets.
10	Inverses	• Find inverses of functions. • Use inverses to reverse actions.
		• Inverse practice worksheets.

Section 1 - Building Cities

Recommended Time: 1 Class Period

OVERVIEW

In this section, students will build their first cities. These cities are representations of functions and are created with specific rules, named "Feature A" and "Feature B" here. Students begin by constructing 2 cities with a team and share them with the class, learning and reinforcing their learning.

Objectives: Students will be able to…	**Assessments**
Build a "city" following specific criteria.	Cities constructed by teams (Activities 1.2)
Create accurate models of cities.	City diagrams (Activities 1.3 and 1.4)
Evaluate whether a city meets specific criteria.	Question answers (Activities 1.3 and 1.4)

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.C** Analyze functions using different representations.

Day 1 PROCEDURE

Opening

1. Activity 1.1 – CLASSWORK – Discussion: Properties of a City (p. 3)
   a. Separate students into teams.
   b. Introduce city building materials.
   c. Introduce Features A and B required for making correct cities. See p. 6 for details.

Body

2. Activity 1.2 – TEAM WORK – Build a City (p. 3)
   a. Teams build cities that have 3 buildings in them.
   b. Teams share out. Note flaws in cities.
3. Activity 1.3 and 1.4 – TEAM WORK – City Diagrams Practice (p. 4-5)
   a. Students diagram each group's cities.

Closing

4. Review Features A and B as a class.

PREPARATION

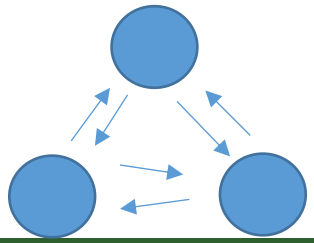
Materials Used	**Resources Used**
• Laminated, numbered sheets for labeling cities. • Uncolored roads (made of duct tape, oil cloth, poster board strips, etc.) with a one-way arrow on them.	•

MODIFICATIONS/EXTENSIONS

- Challenge students to predict how many ways they can build a 3-building city.
- Road Coloring Quiz #1 or Road Coloring Quiz #2 may be used to assess skills learned in this section.

ADDITIONAL NOTES

1. Be sure to spend enough time preparing the physical space where students will be building their cities.
2. Read through the instructions on page 3. Post the **Properties of a City** on a whiteboard, Promethean board, chart paper etc. so that students can easily access them. The teacher should emphasize that roads should be connected to buildings and not to other roads.
3. Guide students to gather necessary materials and begin Activity 1.2 (Page 3). Allow groups to come up with a name for their city. Be sure that the city constructed below comes up in the student examples. The teacher may need to add it if no student in the class produces it.



4. Prompt groups to complete Activity 1.3 (Page 4) by inspecting the different cities that other groups have built and recording them on their worksheet. They should be able to state whether each group created something that did or did not satisfy the properties of a city and why.

Section 2 - Naming the Roads

Recommended Time: 1 Class Period

OVERVIEW

In this section, students further their understanding of the representation of functions by "coloring" their roads. Students will turn their previously built cities into "touring cities" by coloring each of the roads either red or blue, then evaluating each other team's cities and road colors.

Objectives: Students will be able to…	**Assessments**
Correctly color cities.	Colored cities (Activity 2.2)
Represent colored cities with diagrams.	City diagrams (Activity 2.2)
Evaluate whether cities follow road coloring rules.	Questions (Activity 2.2)

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.

PROCEDURE

Opening

1. Activity 2.1 – TEAM WORK – Discussion: How to Color Roads (p. 6)
   a. Introduce road coloring and the rules.
   b. Introduce the term "touring city", or a city with colored roads.

Body

2. Activity 2.1 – TEAM WORK – Coloring Your Roads (p. 6)
   a. Teams color the roads from their cities in Activity 1.2. (p. 4)
3. Activity 2.2 and 2.3 – INDIVIDUAL WORK – Touring Cities (p. 7)
   a. Students copy the colored roads from teams' cities and decide whether they are correctly done.

Closing

4. Share Activity 2.2 and 2.3 answers and discuss

PREPARATION

Materials Used	**Resources Used**
• COLORED roads (red and blue) made of duct tape, oil cloth, poster boards strips, etc. that have one-way arrows drawn on them • RED and BLUE pen, marker, etc. for each student	• Activity 2.2 and 2.3 projected on screen

MODIFICATIONS/EXTENSIONS

- Why do students think there are eight spaces provided for the answers for correctly colored 3-building cities?

ADDITIONAL NOTES

1. Students will need to recreate their previously designed 3-building cities.
2. Read page 6 and ensure that students are clear on the definition of a correctly colored city. Post the qualifications for a correctly colored city (every building has exactly one red road and one blue road leading away from it) on a whiteboard, Promethean board, chart paper, etc. where it is easily visible to students.
3. When students incorrectly color their city, the teacher can tell the students that "there is no money to build new roads, we can only change the name (color) of the roads". Students will then simply switch the two roads coming out of one building in order to preserve the original design (R to B and B to R).
4. Teacher should assist students in completing a few different answers to Activity 2.2 (page 7). Students can then complete as many problems as possible in Activities 2.2 and 2.3. Remind them to be mindful of the features of a correctly colored city posted on the board and found on page 6. Students can share answers with the class on the board.

Section 3 - Solving the Puzzle

Recommended Time: 1 Class Period

OVERVIEW

In this section, students act out the Road Coloring problem in front of the class. The class works on diagramming and recording skills, while also thinking through another function representation. The goal of the activity is to come up with a set of directions to get everyone to the same building of the city at the same time. All of the acting out will be helpful while exploring representations of functions.

Objectives: Students will be able to…	**Assessments**
Create a set of directions, if one exists, that will get all the people in a city to the same building, at the same time, using the cities that were designed previously.	Class discussion and student answers (Activity 3.2)
Evaluate sets of directions for correctness.	Questions (Activities 3.2 and 3.3)

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation**.**
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.

PROCEDURE

Opening

1. Activity 3.1 – CLASSWORK – Discussion: Defining the Puzzle (p. 9)
   a. Review touring cities.
   b. Define the puzzle and how to go about solving it with students. See p. 9.

Body

2. Activity 3.2 – INDIVIDUAL WORK – Puzzle Solving Practice (p. 10)
   a. Each team acts out the and tries to solve the puzzle for their city and have students follow along and record information on p. 10

Closing

3. Activity 3.3 – TEAM WORK – Discussion Questions (p. 11)
   a. Students answer questions about the sets of directions as a team.

PREPARATION

Materials Used	**Resources Used**
• RED and BLUE pen, marker, etc. for each student	•

MODIFICATIONS/EXTENSIONS

None

ADDITIONAL NOTES

1. Students will need to recreate their previously designed (and correctly colored) 3 building city.
2. Read the instructions for solving the puzzle on page 9 and ensure that students understand directions. Highlights include:
   a. Each student will start in a different building.
   b. Each student MUST move when given the instruction.
   c. If everyone ends in the same building at the same time after the set of instructions are given, they have solved the puzzle.
3. Each student will then record whether each team was able to solve the puzzle for each correctly colored city and record the results in Activity 3.2 (page 10).
4. Students/teams will need extra time in working through Activity 3.3. The conversations are important to understanding, and some teams may need more time than others.

Section 4 - Models and Representations

Recommended Time: 1 Class Period

OVERVIEW

In this section, students learn about the Road Coloring Problem. Students learn about modeling the problem, and start to build representations of functions. Specifically, they learn the terminology of "directed graph" and "arrow diagrams."

Objectives: Students will be able to…	Assessments
Create directed graph models of cities.	Activity 4.3
Record movements in the Road Coloring Problem.	Activity 4.3
Create arrow diagrams of cities.	Activity 4.5 and 4.6
Draw the arrow diagrams and road subgraphs for the red and blue roads given the cities.	Activity 4.8

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation**.**
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.

PROCEDURE

Opening

1. Activity 4.1 – TEAM WORK – Create a Representation (p. 14)
    a. Teams create their own representation of a city.

Body

2. Activity 4.2 – CLASS WORK – Class Discussion: The Road Coloring Problem (p. 14-16)
    a. Introduce the idea of simultaneous movements of people in the cities.
3. Activity 4.3 – INDIVIDUAL WORK – Solving the Road Coloring Problem (p. 17)
4. Activity 4.4 – CLASS WORK – Class Discussion: Arrow Diagrams (p. 18-20)
    a. This is the introduction of arrow diagram representations of the cities.
5. Activites 4.5/4.6 – INDIVIDUAL WORK – Arrow Diagrams and Directed Graphs (p. 21-22)
6. Activity 4.7 – CLASS WORK – Class Discussion: Road Subgraphs (p. 23)
    a. Students learn to look at a single color of arrows at a time.
7. Subgraphs become important as students learn about more representations of functions.

Closing

8. Activity 4.8 – INDIVIDUAL WORK – Directed Graphs from Arrow Diagrams (p. 24)
    a. Depending on time, this could very likely be homework.

PREPARATION

Materials Used	Resources Used
• RED and BLUE pen, marker, etc. for each student	• Online or reference materials used

MODIFICATIONS/EXTENSIONS

It's recommended that struggling students have their work checked off before continuing in this lesson.

ADDITIONAL NOTES

1. Emphasize the importance of knowing mathematical terminology.
2. While reading pages 14-16 together, the city on page 14 should be built in front of the classroom. Students should be assigned to play the roles of Candice, Miguel, and Marcus. Students should actually do the movements as the commands are called out.
3. When reading page 15, ask students if they know of any other areas that use the concept of synchronizing (swimming, setting the time of watches, etc.) to aid in understanding.
4. Activity 4.3 (page 17) should be completed for each team's correctly colored city and synchronizing set of directions.
5. As students are reading page 18, students should recognize that it does not matter who is standing in any building. It is only the function (the action or movement of the people) on the roads that matters.
6. On page 19, be sure to emphasize that the arrows must always point to the "go to" buildings.

7. The teacher may want to complete one problem from 4.3 and 4.5 together and assign the rest as individual work. Once students understand the concept of Arrow Diagrams, they generally can complete the activities fairly quickly – so plan accordingly.
8. Note that subgraphs can be thought of as a part of the original graph.

Section 5 – Using Representations and the Cartesian Plane

Recommended Time: 1 Class Period

OVERVIEW

In this section, students learn additional representations of the cities. Subgraphs are representations that only use one color of arrows. Ordered pairs may be familiar to students, however they will learn a new type of information that ordered pairs can convey.

Objectives: Students will be able to…	**Assessments**
Draw the arrow diagrams and list the ordered pairs and plot the points on a Cartesian plane for the functions of the blue and red roads.	Activity 5.2
Draw the associated arrow diagrams and produce the directed graph for the city given the ordered pairs for the red and blue roads.	Activity 5.2
Complete multiple representations of cities given information in one representation.	Representations in Activity 5.4 (p. 30)
Analyze representations of cities and apply that information to the Road Coloring Problem.	Answers to questions in Activity 5.4 (p. 31-32)

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.
- **HSN-Q.A** Reason quantitatively and use units to solve problems.
- **HSS-ID.A** Summarize, represent, and interpret data on a single count or measurement variable.

PROCEDURE

Opening

1. Activity 5.1 – CLASS WORK – Class Discussion: Arrow Diagrams and Ordered Pairs (p. 25-26)
    a. This may review on ordered pairs, but it's important to convey the information that they represent.

Body

2. Activity 5.2 – INDIVIDUAL WORK – Arrow Diagrams and Ordered Pairs Practice (p. 27-28)
3. Activity 5.3 – CLASS WORK – Class Discussion: Representation Review (p. 29)
    a. It may be beneficial to project images of the representations that the students explored in previous sections.
4. Activity 5.4 – TEAM WORK – Understanding Multiple Representations (p. 30-32)
    a. Students should get their representations checked off by the teacher before answering questions or they may struggle through the rest of the questions.

Closing

5. Review answers from Activity 5.4 as a class.

PREPARATION

Materials Used	**Resources Used**
• RED and BLUE pen, marker, etc. for each student	•

MODIFICATIONS/EXTENSIONS

- If students have the background, a side-bar discussion of subsets might happen here.
- Students can find solutions (if they exist) to the cities previous activities.
- Students can provide other representations to cities seen in previous sections.

ADDITIONAL NOTES

1. Emphasize that the first coordinate is the "leave from" building number and the second coordinate is the "go to" building number.

2. Be sure that students understand that the "leave from" is the x-coordinate and the "go to" is the y-coordinate of the graphed points.
3. Teachers should make sure that students label their points on the graph with the correct ordered pairs.
4. In trying to solve the city in Activity 5.4, students will discover that there are two red roads leaving building two, which means there are two ways to leave from building two when red is called. The second error is that there is no blue road leading out of building two, so there is no way to leave building two when blue is called.
5. All of the representations have their own way of saying that "there is no blue road leaving Building 2." Students will most likely not see that there is a difference in how to describe that something is wrong in each of the representations – encourage students to think of what features of each representation indicate that something is wrong (i.e. coordinate planes cannot have more than one point in any of the "leave from" buildings).

Section 6 – Functions

Recommended Time: 1 Class Period

OVERVIEW
In this section, students are introduced the formal mathematical concept of a "function". Using the representations that they've learned and become familiar with, students will explore domains and ranges and learn to evaluate whether what they are looking at is a function. Finally, students learn the Vertical Line Test to determine when a relation isn't a function.

Objectives: Students will be able to…	Assessments
Determine whether a representation of a city represents a function.	Question on Activity 6.2. (p. 34)
Write the domain and range of a function, given a representation.	Last column on Activity 6.2. (p. 35)

STANDARDS
- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.

PROCEDURE

Opening
1. Review the representations of cities that they've used so far.

Body
2. Activity 6.1 – CLASS WORK – Class Discussion: Functions (p. 33-34)
    a. Emphasize precise mathematical language.
3. Activity 6.2 – TEAM WORK – Function Practice (p. 34)
    a. Share answers from Activity 6.2 with the class.
4. Activity 6.3 – CLASS WORK – Class Discussion: The Vertical Line Test

Closing
5. Activity 6.4 – TEAM WORK – Vertical Line Test Practice

PREPARATION

Materials Used	Resources Used
• Projection of representations of cities.	•

MODIFICATIONS/EXTENSIONS
Students can determine if cities seen in previous sections are functions.

ADDITIONAL NOTES
1. The concept of function has been explored throughout this module. Now a formal discussion of a function occurs. While reading pages 33 and 34, have students compare and contrast their experience with function which meant that "every building in the city must have exactly one red road and one blue road leading away from it" to the traditional definition of a function as "a relation that for every input 'x' there can only be one output 'y'".
2. Remind students that adults may not understand the vernacular (slang, common language) that they use with their peers. Connect their communication experiences to the ways that mathematicians communicate and are often misunderstood. Stress the importance of using precise mathematical language when they express their ideas in discussions of the math.
3. It is helpful to project the examples from 6.3 and demonstrate using some sort of tool (paper or meter stick) to show how it might look without actually drawing lines in.

Section 7 - Matrix Representations

Recommended Time: 1 Class Period

OVERVIEW
In this section, students begin by trying to come up with their own representation of cities. They are then introduced to matrix representations of functions and practice going back and forth between matrices and other representations of the cities.

Objectives: Students will be able to…	Assessments
Create a matrix representation of a city.	Matrices in Activity 7.3.

STANDARDS
- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.
- **HSN-VM.C** Perform operations on matrices and use matrices in applications.

PROCEDURE

Opening
1. Activity 7.1 – TEAM WORK – A New Representation (p. 38)

Body
2. Activity 7.2 – CLASS WORK – Class Discussion: Matrix Representations of Functions (p. 39-40)
    a. Be sure to note that all spaces must be filled with a number. Leaving spots blank or putting symbols like x's or check marks is a common misconception.
3. Activity 7.3 – TEAM WORK – Matrix Practice (p. 41-42)

Closing
4. Review answers to Activity 7.3 as a class.

PREPARATION

Materials Used	Resources Used
• RED and BLUE markers	• None

MODIFICATIONS/EXTENSIONS
This may be a good place to give a formal or informal assessment of all the representations studied up to this point. Aside from function notation and equations, students pretty much have studied all the representations and can determine whether or not a city is a function based on its arrow diagrams, ordered pairs, coordinate planes, subgraphs, and now matrices.

ADDITIONAL NOTES
1. Be sure to clearly explain teacher expectations for what their representations should look like (i.e. they are not using the materials to simply recreate a representation that has been done, like using popsicle sticks and poker chips to build a directed graph). Students may need to see an example first, but be sure to reiterate that students are not to simply imitate your example.
2. Presenting their unique representations is an important part of this activity, so maybe have students create their own arrow diagram (instead of using the one in the text) and then post their unique representations and have other students interpret the meaning. Students can draw a picture of their unique representation on poster paper and present that way, or all students could walk around the room (I.e. gallery walk) and examine the unique representations that other students made.
3. Students should be able to name the location of each entry by row and column. For example, in the matrix [0 1 0], the entry 1 is in Row 1 and Column 2.
4. A student discussion of the real world examples of matrices should be held. Teacher will have to lead this discussion and possibly start by giving some examples (i.e. school master schedule, computer programming, etc.).
5. Teacher may want to do some examples together.
6. Remind students that an entry of "0" means there are no roads connecting the two buildings and an entry of "1" means there is one road connecting the two buildings.

Section 8 - A New Operation

Recommended Time: 1 class period

OVERVIEW

In this section, students learn to combine multiple arrow diagrams to “compose” the functions. They also learn to use these composed functions as shortcuts to follow movements in the cities. Finally, they learn to multiply matrices by converting them to arrow diagrams, composing them, and converting them back to matrices.

Objectives: Students will be able to…	Assessments
Represent combined effect of two instructions in a single arrow diagram.	Activities 8.1 (p. 43-45) and 8.2 (p. 46-48)
Compose arrow diagrams to a single diagram.	Activities 8.2 (p. 46-48) and 8.5a (p. 53)
Multiply matrices using arrow diagrams.	Activities 8.4 (p. 52) and 8.5b (p. 54)

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.
- **HSF-BF.A** Build a function that models the relationship between two quantities.

PROCEDURE

Opening

1. Activity 8.1 – CLASS WORK – Class Discussion: Function Composition (p. 43)
   a. On page 37, have students create arrow diagrams in Table 8.1 from the city in Figure 8.1a.

Body

2. Activity 8.1 – CLASS WORK – Class Discussion: Function Composition (p. 43-45)
   a. Work through compositions together on a whiteboard or with a projected version.
3. Activity 8.2 – TEAM WORK – Combining Arrow Diagrams (p. 46-48)
   a. Periodic class pauses to go over answers together may help.
4. Activity 8.3 – CLASS WORK – Class Discussion: Multiplying Road Matrices (p. 49-50)
5. Activity 8.4 – TEAM WORK – Matrix Multiplication (p. 52)

Closing

6. Activity 9.5 – INDIVIDUAL WORK – Function Composition and Matrix Multiplication (p. 53-54)
   a. Likely will be homework.

PREPARATION

Materials Used	Resources Used
• Red and Blue Markers	•

MODIFICATIONS/EXTENSIONS

ADDITIONAL NOTES

1. Working with students on the board during Activities 8.1 and 8.3 is highly recommended.
2. Let the student calling out the two instructions take ownership for what choices are made for the first and second instructions called.
3. Students should be advised to record their observations carefully for the two instructions called and then for the combined effect (which they will soon learn is a composition of two functions).
4. Students will become easily confused by the composition of two identical commands (i.e. R-R or B-B) and should be reminded that we are only looking at where the character ultimately starts and ends, forgetting where the roads lead in and out of in that second arrow diagram.
5. Be sure to ask, "Did everyone get to the same building at the same time?"

Section 9 – Function Notation

Recommended Time: 1 Class Period

OVERVIEW

In this section, students learn a formal, mathematical representation of what they have been doing throughout the unit. In using function notation, they connect their Road Coloring work with their more formal mathematical work and develop connect the language they've used with functions they will continue to use. Finally, they learn to represent function composition using their new function notation.

Objectives: Students will be able to…	Assessments
Use function notation to describe cities from multiple representations.	Activity 9.2
Represent the composition of functions using function notation.	Activity 9.4

STANDARDS

- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.B** Interpret functions that arise in applications in terms of the context.
- **HSF-IF.C** Analyze functions using different representations.
- **HSF-BF.A** Build a function that models the relationship between two quantities.

PROCEDURE

Opening

1. Review function composition Activities 8.5a and 8.5b.

Body

2. Activity 9.1 – CLASS WORK – Class Discussion: Function Notation (p. 55)
   a. The teacher should go through multiple examples with the class. This can be confusing, but seeing several examples can help solidify the skill.
3. Activity 9.2 – TEAM WORK – Function Notation Practice (p. 56)
   a. Go over answers as a class.
   b. Groups finishing early can always go back to other examples throughout the student workbook.
4. Activity 9.3 – CLASS WORK – Class Discussion: Composing Function Notations (p. 57)

Closing

5. Activity 9.4 – TEAM WORK – Function Notation Composition Practice (p. 58)
   a. Discuss answers if there is time.

PREPARATION

Materials Used	Resources Used
• None	• None

MODIFICATIONS/EXTENSIONS

Students finishing early can write function notations for examples throughout the student workbook.

ADDITIONAL NOTES

1. Students will make a variety of mistakes throughout this lesson, particularly in looking at different representations of functions. Make sure to share examples with the class as much as possible, as it will reinforce correct function notation with students.
2. Be sure to use a variety of function representations to use as the pre-function notation forms, as any misconceptions from previous sections will become evident in this section.

Section 10 – Inverses

Recommended Time: 1 Class Period

OVERVIEW
The final section of the Road Coloring curriculum has students explore inverses of functions as "reversals" of the action originally taken. Students learn that inverses work the same way as the original function by creating them in each representation that they've previously learned.

Objectives: Students will be able to…	Assessments
Represent inverses of functions using multiple representations.	Activity 10.2
Determine whether the inverse of a function is also a function.	Activity 10.4
Justify whether an inverse is a function.	Activity 10.4

STANDARDS
- **HSF-IF.A** Understand the concept of a function and use function notation.
- **HSF-IF.C** Analyze functions using different representations.

PROCEDURE

Opening
6. Review function representations with a sample city.

Body
7. Activity 10.1 – CLASS WORK – Class Discussion: Inverses (p. 59)
   a. The teacher should pause after each bullet point to allow students to follow along, then review answers as a class.
8. Activity 10.2 – TEAM WORK – Inverse Practice with Multiple Representations (p. 60-61)
   a. Go over answers as a class.
9. Activity 10.3 – CLASS WORK – Class Discussion: Domains and Ranges of Inverses and if they are Inverse Functions (p. 62)

Closing
10. Activity 10.4 – TEAM WORK – Inverse Function Practice (p. 63)
    a. Discuss answers if there is time.

PREPARATION

Materials Used	Resources Used
• None	• None

MODIFICATIONS/EXTENSIONS
Students finishing early can write function notations for examples throughout the student workbook.

ADDITIONAL NOTES
1. In Activity 10.1, be sure to pause for students to think about their answers for each bullet. Not taking the time to discuss their answers as a class, or at least in groups, could leave students missing the idea of inverses. It may be important to demonstrate reversing arrows by only replacing the head of an arrow.
2. Be sure to use a variety of function representations, as students' comfort with each representation may vary.
3. The idea of switching the domain and range of a function to find the inverse is important. Understanding this concept allows for students to utilize their previous understandings of how to determine if relations are functions.